Housing Investment: What Makes It so Volatile?
Theory and Evidence from OECD Countries*

Quoc Hung Nguyen †

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Abstract

This paper explains how mortgage market liberalization can introduce greater volatility in the housing market, which is a stylized fact documented from OECD countries, with a DSGE model where households face a credit constraint and housing is used as collateral. The housing collateral constraint creates a link between the housing market and borrowing capacity, a link that amplifies the response of housing demand to technology shocks and strengthens in economies with more liberalized mortgage markets. The calibrated model is able to explain about 90 percent of housing investment volatility in the UK.

JEL Classification: E22, E32, F34, F41

Keywords: Housing Investment, Collateral Constraint, Mortgage Markets

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1 Introduction

“Why does housing investment, which is already highly volatile from the business cycle perspective, become even more volatile in economies with more liberalized mortgage markets?” The questions are of interest because of several reasons. First, the global financial meltdown has generated wide interest in the impact of deregulation in the mortgage market on the housing sector and the overall business cycle. Second, variance in the housing sector accounts for a significant share of variance in output in many economies and housing investment offers the best early warnings of an oncoming recession among GDP components. \(^1\) Therefore, it is important to understand the dynamics of housing investment in order to control business cycles. Third, the high volatility of housing investment in the US has not obtained enough satisfactory explanations. To my knowledge, the exception is Davis and Heathcote (2005), which explains the high volatility from the supply side and does not address the mortgage market.

Data from OECD countries indicate that the high volatility of housing investment is not a distinguishing feature of the U.S. Across 17 OECD countries, housing investment is on average about five times as volatile as output. More interestingly, housing investment tends to be more volatile in Anglo Saxon and Nordic economies where the mortgage market had been significantly deregulated before the financial crisis. In particular, in these economies, the standard deviation of housing investment is about two times as large as non-housing investment and is about six to seven times as large as output. This positive association between housing investment volatility and the degree of mortgage market liberalization \(^2\) contradicts the opinion, which was widely spread among academics and policy-makers before the 2007-8 financial meltdown, that credit market innovation should enhance market stability.

Standard business cycle models with perfect credit markets are at odds with these facts. First, these models are unable to explain the positive correlation between housing investment volatility and mortgage market development since the degree of mortgage market liberalization should be immaterial under a perfect credit market assumption. Second, these standard models are also at odds in reconciling the high volatility of housing

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\(^1\) Housing demand shocks account for 20-25% of variance in GDP in the U.S and Japan (IMF, 2008) and in the past 60 years, eight out of ten recessions in the US were preceded by substantial problems in housing.

\(^2\) The positive correlation is not limited by cross-country evidence but is also reflected by time series data. The volatility of housing investment relative to output has also risen along dramatic mortgage market innovation.
investment. I shows in this paper that a quantitative two-sector model with free borrowing fails generating a realistic volatility of housing investment.

To explain the aforementioned stylized facts, I develop a Dynamic Stochastic General Equilibrium (DSGE) model with a borrowing constraint. Specifically, I consider a limited obligation environment in which borrowers do not repay unless debts are secured by collateral and housing plays the collateral role. The housing collateral constraint is inspired by the fact that the major part of household borrowing has been in the form of collateralized debt. For example, the shares of mortgage debt in total outstanding household debt are about 80% in the US. There is also evidence of borrowers’ limited obligations. For instance, when the subprime mortgage market worsened, many borrowers just walked away from their housing collateral without any further obligations. Housing collateral is rationalized by the fact that housing is a very good store of value and an important component of wealth for most households. In the US, the value of housing structures excluding land is similar to the combined value of private non-housing structures and equipment, similar to annual GDP, and three times as large as the total stock of all other consumer durables.

The mechanism through which a housing collateral constraint affects the dynamics of housing investment goes as follows. Households are impatient because of borrowing constraints. Therefore, when housing is used as collateral, housing collateral value that defines households’ borrowing capacity is endogenously determined by the housing market. As a result, the housing collateral constraint creates a link between the housing markets and borrowing capacity, a link that amplifies the response of housing demand to shocks. Intuitively, in a good time when a positive shock hits the economy, households invest more in housing not only because housing is a normal durable consumption good but also because they obtain the benefits of relaxing borrowing constraints with more collateralizable housing assets. Once the demand for housing increases the housing price goes up, which in turn raises the collateral value, hence allowing impatient households to borrow more to consume and further fueling the demand for housing. This is the financial accelerator effect or financial multiplier of housing collateral constraint, which helps explaining the highly volatile behavior of housing investment.

The housing collateral constraint can also account for the positive correlation between housing investment volatility and the degree of mortgage market liberalization. In other words, mortgage market innovation and liberalization may introduce instability in the housing market. The underlying reason is that in economies with more flexible and liberalized mortgage markets, credit-constrained households can borrow a higher amount.
for the same value of collateral and easily withdraw equity from increased collateral for consumption. As a result, mortgage market innovation and liberalization intensify the collateral role of housing, encouraging credit constrained households to purchase more houses in good times. Besides, these innovation and liberalization also strengthen the link between the housing market and the consumption decisions, hence creating a stronger borrowing-consuming spiral.

This work first is related to the business cycle literature that incorporates the housing sector. The literature often documents regularities, distinguishes housing investment from its non-housing counterpart, and attempts to explain the co-movement between the two types of investment. These authors, however, often have difficulty in accounting for the relatively high volatility of housing investment. For example, Baxter (1996) finds that consumption of durables that include housing is less volatile than business investment; Fisher (1997) is unable to generate household investment more volatile than business investment for all specifications. Davis and Heathcote (2005) explain the co-movement and the high volatility by building a model where housing and the other sectors all use three intermediate goods but in different proportions. The high volatility is the result of the calibration that the housing construction sector uses a relatively higher proportion of intermediate goods which are more volatile. It is, however, unclear whether their estimate of the Solow residual of housing construction production originates only from productivity shocks or the mixed equilibrium outcome of supply and demand or both in the housing sector. By contrast, this paper explains the high volatility from the demand side of the housing sector.

The housing collateral constraint in this paper originates from the seminal work of Kiyotaki and Moore (KM) (1997) and Kocherlakota (2000). These authors show that collateral effects can be a powerful propagation mechanism by which relatively small, temporary shocks can generate large, persistent fluctuations in output and asset prices. Campbell and Hercowitz (CH) (2005) develop a one-sector real business cycle model to address the impact of credit market innovations on macroeconomic volatility. Their

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3Greenwood and Hercowitz (1991) and Baxter (1996) assume reversibility between residential and business capital and also highly correlated productivity shocks between two sectors. Fisher (1997) assumes complementarity between the household and business capital in goods production, specifically, a nonlinear function for transforming output into non-durable consumption goods, new consumer durables, and new physical capital. Chang (2000) argues that if there are adjustment costs in capital accumulation and substitutability between leisure time and durable goods in home production, then when households work more in periods of high productivity they also demand more durables. See Charles Leung (2004) for further details.
mechanism is through labor supply: less tight collateral constraints weaken the connection between constrained households’ housing investment and their hours worked. Iacoviello (2005) incorporates the New Keynesian monetary policy framework into the work of KM. Collateral effects enable his model to match the positive response of spending to a housing price shock. Calza et al. (2007) extend Iacoviello’s work to allow production of new housing and endogenous asset price movement. They also model institutional features of the mortgage market and argue that the correlation between consumption and house prices increases with the degree of mortgage market development, and the transmission of monetary policy shocks to consumption and to housing prices is stronger in countries with more developed mortgage markets. More recently, Monacelli (2008) argues that introducing a collateral constraint into the New Keynesian framework can reconcile the co-movement of durable and non-durable spending in response to monetary shocks.

My work differs from these in many key aspects. Unlike the CH work, it develops a two-sector model and incorporates asset price movement to explore the amplification mechanism of collateral effects. As a result, my paper is able to produce the positive association between the degree of mortgage market liberalization and housing investment volatility. In contrast with Iacoviello (2005), Calza et al. (2007), and Monacelli (2008), which are New Keynesian models with nominal sticky prices and nominal debt, models in this paper have flexible prices and real debt and focus on the technology shock.

Another feature of the model in this paper is openness with exogenous interest rate and endogenous capital accumulation. Openness with exogenous interest first is to create the positive co-movement between housing investment and non-housing investment, which is different with Greenwood and Hercowitz (1991) and Baxter (1996), Fisher (1997), and Chang (2000) as mentioned above. Openness also allows domestic agents to access international credit markets, making it possible to address the model without household heterogeneity. Finally, capital accumulation better characterizes the dynamics of the current account. Notably, compared to small open economy models without financial frictions, which often have difficulty in generating the counter-cyclicality of the current account (Backus et al. (1992) and Mendoza (1991)), the collateral constrained model produces a significant counter-cyclicality.

This paper is organized as follows. Section 2 describes data, particularly two mortgage market depth indicators, and documents stylized facts about housing investment and its association with mortgage market depth. Section 3 explains the empirical findings using

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4The existing literature often considers a closed economy model with heterogeneous agents where patient savers lend to impatient borrowers
a basic model with a borrowing constrained representative household. Section 4 extends
the basic model to include heterogeneous households, discusses the model’s dynamics, and
calibrates it for the U.K. Section 5 concludes.

2  Stylized Facts

This section documents major stylized facts of the housing sector with an emphasis on
housing investment and the mortgage market in 17 advanced OECD countries from Q1-
1980 to Q3-2007. The choice of 17 OECD countries is mainly based on the availability
of data. They are Australia, Austria, Belgium, Canada, Denmark, Finland, France,
Germany, Italy, Japan, the Netherlands, New Zealand, Norway, Sweden, the U.K, and
the U.S. I choose post 1980s period since most deregulation in the mortgage market began
in the early 1980s.

2.1  Data

All time series data are quarterly, except Germany’s annual and Italy’s half-year house
prices. House prices are mainly provided by the Bank for International Settlements, and
other missing values are filled and updated via Datastream. Real house prices are then
obtained by deflating nominal house prices with the consumer price index (CPI). Housing
investment, which is residential investment, non-housing investment, aggregate invest-
ment, and GDP are in real values and obtained via Datastream and OECD Statistics.

Two specific indicators are used to measure the degree of mortgage market liberal-
ization in these OECD countries. The first one is a synthetic mortgage market index
constructed by the IMF. 5 The second measure is the ratio of total outstanding amount of
mortgage debt over GDP, the mortgage-debt-to-GDP ratio or the mortgage depth, which
is often used in literature. 6

In particular, thanks to deregulation and innovations in the mortgage market, most
of advanced OECD countries had moved toward more liberalized housing finance mar-
kets. Nonetheless, there were still significant cross-country differences in the degree of

5 They are taken from Table 3.1 of Chapter 3 of IMF World Economic Outlook (WEO) April 2008:
“The changing housing cycle and the implications for monetary policy”.
6 For example: Warnock and Warnock (2008) use this ratio or maximum possible of this ratio to
measure mortgage market depth or market size. The data for mortgage-debt-to-GDP ratio (2001-2006
average) for all countries except New Zealand is taken from the IMF. Data for New Zealand are taken
from Warnock (2008).
mortgage market liberalization in terms of legal procedures, regulatory structures, and securitization. The cross-country differences in mortgage market liberalization and development are reflected through: (1) The typical ratio of a mortgage loan to property’s value or loan-to-value (LTV) ratio and the standard length of mortgage loans; (2) The ability to make home equity withdrawals and to prepay mortgages without a fee; (3) Developments of secondary markets for mortgage loans or the degree of securitization. These differences then imply different households’ access to housing-related financing in each country. To summarize cross-country differences in mortgage market liberalization, one synthetic mortgage market index is constructed. The index lies between 0 and 1, with higher values indicating more liberalized mortgage markets with easier household access to mortgage credit. This IMF’s mortgage market index (henceforth MMI) and the mortgage-debt-to-GDP ratio or the mortgage depth (henceforth MD) are closely positively correlated, i.e., the economies with a higher mortgage market index often have a bigger or deeper mortgage market size (Figure 3). Figures 1 and 2 show evidence that there are significant differences in the degree of mortgage market liberalization and development and mortgage size, even among advanced OECD countries.

[Figure 1 about here.]

[Figure 2 about here.]

[Figure 3 about here.]

Since the IMF’s index is a one-period time indicator, which may capture precisely only the current degree of mortgage market development, I extend data for the second indicator, the mortgage-debt-to-GDP ratio, to the last 10 years in order to examine the development of the mortgage market over time. 7 Figure 4 suggests that the degree of mortgage market depth has been increasing for most of these countries but the rank remains the same, i.e., those countries that currently have deeper mortgage markets also possessed deeper ones in the 1990s. Therefore, I conclude that the IMF’s index reflects the comparative degree of mortgage market development, at least from the 1990s.

[Figure 4 about here.]

7Although some countries like the U.S, U.K and Australia have data before 1997, I could not find longer data for many European countries.
2.2 Stylized Facts

The first regularity of the housing sector in OECD countries is that real housing prices are significantly pro-cyclical with real GDP, which is contrary to the counter-cyclicality of non-housing investment’s real prices (The 2nd Column of Table 1).  

Secondly, the 3rd and 4th Column of Table 1 present evidence that housing investment co-moves with non-housing investment and output in most advanced OECD countries. The pro-cyclicality of both real housing prices and housing investment makes it challenging for those models that try to explain the high volatility of housing investment from supply side, particularly the housing sector specific productivity shocks.

Housing investment, however, differs from its non-housing counterpart in terms of volatility and cross-country dispersion. According to Table 1 (Column 10 and 11), the standard deviation of housing investment relative to GDP is not only significantly higher than that of non-housing investment but also varies widely across countries. The former ratio ranges from 2.56 in Italy to 6.67 in the U.S, whereas the latter ratio is stable at 3.8. The F-test for variances of the two groups is rejected with significant level (p-value is 4%) and the t-test for equality of the two ratios is strongly rejected (p-value=0.2%). I obtain the same conclusions when comparing the housing investment with aggregate investment: housing investment is, on average, much more volatile and varies widely across countries than aggregate investment.

Related to the mortgage market, Figures 5 and 6 first show significant positive correlations between output volatility and the mortgage market indicators. Moreover, Figures 7 and 8 present evidence that the volatility of housing investment relative to that of output is higher in economies with more liberalized and deeper mortgage markets, i.e., economies with higher mortgage market indices and larger mortgage market size, while there is no significant correlation between the volatility of non-housing investment and degree of mortgage market development (Figure 9 and 10). In other words, these figures show that while output tends to be more volatile in economies with more liberalized and deeper mortgage markets, housing investment is still more volatile. Therefore the volatility of housing investment to output significantly increases in these countries.

8As documented by Greenwood et al. (1997) and Fisher (2006), the real non-housing investment price measured by the business equipment deflator divided by consumption deflator is significantly counter-cyclical with GDP: The unconditional correlation for the U.S economy is -0.54.
Finally, I explore housing investment from a historical perspective. Since most deregulation and innovation in the housing finance system began in the early 1980s, and it is evident that the current system has been much developed and liberalized compared to that in the early stage of deregulation, I divide samples into 2 equal periods: prior and post Q1-1995. Table 2 first presents evidence that the volatility of output has dropped dramatically over time across advanced OECD countries: post 1995, the average standard deviation of GDP is about two times as low as that prior to 1995. However, the volatility of housing investment has not fallen by that much so that the volatility relative to GDP has risen. In other words, housing investment has become relatively more volatile along with innovations in the mortgage market in these OECD countries.

### 3 Basic Model

To explain these empirical findings, I construct a two-sector DSGE model in which a representative household faces a borrowing constraint and housing plays the collateral role. A two-sector model is necessary to analyze housing that is a durable and non-tradable good as opposed to the non-durable but tradable non-housing good.

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9I use 10 out of 17 countries that have relatively long enough observations before Q1-1995.

10The t-test for the equality of the two ratios of relative volatility is rejected with 10%.
### 3.1 Household

The representative household maximizes its expected lifetime utility defined over random sequences of non-durable consumption goods \((c_t)\), housing services from the housing stock \((h_t)\), and labor disutility \((l_t)\):

\[
U = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t, l_t) \tag{3.1}
\]

The budget constraint of the representative household is given by:

\[
c_t + q_t[h_t - (1 - \delta_h)h_{t-1}] + i^c_t + i^h_t + (1 + r_{t-1})d_{t-1} \leq w_t l_t + r^c_t k^c_{t-1} + r^h_t k^h_{t-1} + d_t \tag{3.2}
\]

\[
i^c_t = k^c_t - (1 - \delta_k)k^c_{t-1} + \frac{\phi_k (k^c_t - k^c_{t-1})^2}{k^c_{t-1}} \tag{3.3}
\]

\[
i^h_t = k^h_t - (1 - \delta_k)k^h_{t-1} + \frac{\phi_k (k^h_t - k^h_{t-1})^2}{k^h_{t-1}} \tag{3.4}
\]

Each period, the household can borrow internationally traded debt, \(d_t\), subject to a constraint described later, at an exogenous real interest rate, \(r_t\). It supplies labor, \(l_t\), at the real wage rate, \(w_t\), and lends sector specific capital, \(k^c_t, k^h_t\), to capital markets at prices \(r^c_t, r^h_t\), where \(k^c_t, k^h_t\) are capital for non-durable and durable production, respectively. The household then spreads its income on non-durable consumption goods, \(c_t\), debt repayment, \((1 + r_{t-1})d_{t-1}\), investments on two types of non-housing capitals \(i^c_t, i^h_t\), and housing investment, \(q_t[h_t - (1 - \delta_h)h_{t-1}]\), where \(q_t\) is real housing prices and \(\delta_h\) is the depreciation rate of housing stock.

In addition to the budget constraint, the representative household faces the following collateral borrowing constraint:

\[
(1 + r_t)d_t \leq \phi E_t(q_{t+1}h_t) \tag{3.5}
\]

which means at any time the amount the household can borrow is limited by the expected future value of his property. As in Kiyotaki and Moore (1997) and Kocherlakota (2000), this borrowing constraint is rationalized by the borrower’s limited obligations. If the household repudiates its debt obligations, the lenders can foreclose the property after paying the transaction costs, \((1 - \phi)E_t(q_{t+1}h_t)\). The parameter \(\phi\), which presents the fraction of collateral value a household can use for borrowing, reflects market liberalization, legal procedures, and regulatory structures or institutional features prevailing in the mortgage market, therefore indicating the degree of the mortgage market flexibility and
development. A higher $\phi$ corresponds to a higher mortgage market index and indicates a more liberalized mortgage market in the model.

In this paper, I specialize preferences as below:

$$U(c_t, h_t, l_t) = \frac{(x_t - \frac{\kappa^{c_t}}{\omega})^{1-\sigma} - 1}{1 - \sigma}$$ (3.6)

$$x_t = [(1 - \gamma)^{\frac{\eta-1}{\eta}} c_t^{\eta} + \gamma^{\frac{\eta-1}{\eta}} h_t^{\eta}]^{\frac{1}{\eta}}$$ (3.7)

This is the GHH preference function introduced by Greenwood, Hercowitz and Huffman (1988). Composite consumption, $x_t$, is the CES function of nondurable consumption, $c_t$, and housing services from the housing stock $h_t$. $\gamma > 0$ is the share of housing services in the composite consumption index. $\eta \geq 0$ is the elasticity of substitution between non-durables and housing services. $\sigma$ denotes the inverse elasticity of intertemporal substitution, $\omega$ determines the elasticity of labor supply, and $\kappa$ determines the amount of leisure in the steady state.

Let’s denote the multiplier on the borrowing constraint at time $t$ by $\lambda_t$ then the first order conditions for the representative household read:

$$U_{ct}[1 + \phi_k (\frac{k_{ct}^c - k_{ct-1}^c}{k_{ct-1}^c})] = \beta E_t\{U_{ct+1}[1 - \delta_k + r_{t+1}^c + \frac{\phi_k}{2} ((\frac{k_{ct+1}^c}{k_t^c})^2 - 1)]\}$$

$$U_{ct}[1 + \phi_k (\frac{k_{ct}^h - k_{ct-1}^h}{k_{ct-1}^h})] = \beta E_t\{U_{ct+1}[1 - \delta_k + r_{t+1}^h + \frac{\phi_k}{2} ((\frac{k_{ct+1}^h}{k_t^h})^2 - 1)]\}$$

$$w_t = -\frac{U_{lt}}{U_{ct}}$$

$$U_{ct} - \lambda_t = \beta E_t\{U_{ct+1}(1 + r_t)\}$$ (3.8)

$$q_t U_{ct} = U_{ht} + \phi \lambda_t E_t\{q_{t+1}\} + \beta E_t\{U_{ct+1}[q_{t+1}(1 - \delta_k)]\}$$ (3.9)

The first two equations are standard optimality conditions for capital with adjustment costs while the third one is a standard labor supply equation. The last two equations present distinguishing features of the borrowing constraint model. Equation (3.8) is a modified Euler equation and is reduced to a standard Euler equation in case of a non-binding constraint, i.e., $\lambda_t = 0$. When the constraint binds, the shadow value of borrowing

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11GHH preferences are commonly used in open economy models. GHH preferences provide a better description of consumption and the trade balance for small open economies than alternative specifications (see, for instance, Correia, Neves, and Rebelo (1995)). See Raffo (2008) for discussions in the context of two-country models.
is positive, $\lambda_t > 0$, so there is an intertemporal distortion in non-durable goods consumption between two different times. In other words, when $\lambda_t > 0$, this equation implies that $U_{ct} > \beta E_t\{U_{ct+1}(1 + r_t)\}$, which means the marginal utility of current non-durable consumption is higher than the marginal gain of shifting one unit of non-durables to the next period. A higher $\lambda_t$ implies a tighter constraint, hence encouraging the household to invest more in collateralizable housing assets in order to relax the constraint, which then enables it to borrow more.

Equation (3.9) is the efficiency condition for the intratemporal choice of durable housing that requires the household to equate the marginal utility of non-durable consumption, weighted by the relative housing prices and adjustment costs, to the marginal utility of housing services. The marginal utility of housing service consists of three components: (i) the direct utility gain of an additional unit of housing; (ii) the marginal gain from relaxing the collateral constraint; (iii) the expected utility derived from expanding future consumption by means of re-selling the amount of housing invested in the previous period. When the constraint doesn’t bind, $\lambda_t = 0$, the distortion component $\phi \lambda_t E_t\{q_{t+1}\}$ vanishes, hence the marginal benefit of housing consists of only terms (i) and (iii), which is the standard intratemporal optimality condition.

After integrating (3.9) forward, I obtain the following demand function for housing:

$$q_t U_{ct} = E_t \left\{ \sum_{j=0}^{\infty} [(1 - \delta_h)\beta]^j U_{ht+j} \right\} + E_t \left\{ \sum_{j=0}^{\infty} [(1 - \delta_h)\beta]^j \phi \lambda_{t+j} q_{t+1+j} \right\}$$

The first term in the RHS of (3.10) is the discounted stream of utility from housing services. The second term is the current and expected benefits from the opportunity to increase consumption by the additional borrowing enabled by increased collateral value. This term depends on the degree of mortgage market liberalization represented by parameter $\phi$, the expected prices of housing, and the tightness of credit constraint $\lambda_{t+j}$. When the constraint doesn’t bind, $\lambda_t = 0$ for all $t$, this term is equated to zero, hence, the weighted marginal utility of non-durable consumption in the LHS equates to the discounted stream of utility from housing services.

### 3.2 Firms

At time $t$, tradable and non-durable goods are produced by a technology that is the Cobb-Douglas function of previously installed capital, $k^{c}_{t-1}$, and labor, $l^{c}_{t}$, as follows:

$$y_t = A_t (k^{c}_{t-1})^{\alpha_c} (l^{c}_{t})^{1-\alpha_c}$$
Output from the tradable non-durable sector then can be used as non-durable consumption, $c_t$, and investments in either type of capital goods, $k_c^t, k_h^t$, and can also be exported as, $tb_t$.

Housing, which is durable but non-tradable, is produced/built using capital, $k_h^t$ and labor, $l_h^t$, as follows:

$$ b_t = A_t(k_{t-1}^h)^{\alpha_h}(l_h^t)^{1-\alpha_h} \quad (3.12) $$

where $A_t$ denotes the aggregate exogenous stochastic productivity shock that follows an AR(1) law of motion: $^{12}$

$$ \log(A_{t+1}) = \rho \log(A_t) + \epsilon_{t+1} \quad (3.13) $$

Optimality conditions for tradable goods firms imply:

$$ w_t = (1 - \alpha_c) \frac{y_t}{l_t^c} = (1 - \alpha_c) A_t \left( \frac{k_{t-1}^c}{l_t^c} \right)^{\alpha_c} \quad (3.14) $$

$$ r_c^t = \alpha_c \frac{y_t}{k_{t-1}^c} = \alpha_c A_t \left( \frac{k_{t-1}^c}{l_t^c} \right)^{\alpha_c} \quad (3.15) $$

Optimality conditions for the construction sector imply:

$$ w_t = q_t(1 - \alpha_h) \frac{b_t}{l_t^h} = q_t(1 - \alpha_h) A_t \left( \frac{k_{t-1}^h}{l_t^h} \right)^{\alpha_h} \quad (3.16) $$

$$ r_h^t = q_t \alpha_h \frac{b_t}{k_{t-1}^h} = q_t \alpha_h A_t \left( \frac{k_{t-1}^h}{l_t^h} \right)^{\alpha_h} \quad (3.17) $$

### 3.3 Equilibrium

Given the interest rate, $r_t$, a competitive equilibrium in this economy is characterized by a sequence of allocations $\{c_t, l_t, h_t, d_t, k_t^c, k_t^h, i_t^c, i_t^h, y_t, l_t^c, l_t^h\}$ and a sequence of prices $\{q_t,w_t,r_t^c,r_t^h,\lambda_t\}$ that satisfy the household and firms optimality conditions, the budget constraint, the binding borrowing constraint, production functions, and the following market clearing conditions.

Labor market clearing:

$$ l_t = l_t^c + l_t^h \quad (3.18) $$

Non-tradable durable housing market clearing:

$$ b_t = h_t - (1 - \delta_h)h_{t-1} \quad (3.19) $$

$^{12}$For simplicity, I assume that an exogenous productivity shock has the same effect on both production sectors or a perfect correlated productivity shock as in Greenwood and Hercowitz (1991). I, however, do not assume reversibility between housing and business capital and housing is produced separately.
Tradable non-durable goods market:

\[ c_t + i^e_t + i^h_t + (1 + r_{t-1})d_{t-1} = y_t + d_t \]  \hspace{1cm} (3.20)

The trade balance, housing investment, and aggregate output can be expressed as:

\[ tb_t = y_t - c_t - i^e_t - i^h_t \]  \hspace{1cm} (3.21)

\[ resi_t = q_t b_t \]  \hspace{1cm} (3.22)

\[ Y_t = y_t + q_t b_t \]  \hspace{1cm} (3.23)

### 3.4 Benchmark: Free Borrowing Economy

For comparison, I also consider a benchmark: a two-sector open economy model with free borrowing. In this economy, the borrowing constraint does not bind so the multiplier \( \lambda_t = 0 \ \forall \ t \). As a result, two optimal conditions for non-durables and housing can be written as:\(^{13}\)

\[ U_{ct}(1 - \phi_d(d_t - \bar{d})) = \beta E_t\{U_{ct+1}(1 + r_t)\} \]  \hspace{1cm} (3.24)

\[ q_t U_{ct} = U_{ht} + (1 - \delta_h) \beta E_t\{q_{t+1} U_{ct+1}\} \]  \hspace{1cm} (3.25)

Hence, the demand function for durable housing becomes:

\[ q_t U_{ct} = E_t\left\{ \sum_{j=0}^{\infty} [(1 - \delta_h) \beta]^j U_{ht+j} \right\} \]  \hspace{1cm} (3.26)

The RHS of equation (3.26) is the shadow value of durable housing. According to Barsky et al. (2007), there are two reasons that keep this value roughly constant against moderate-lived shocks. First, durable housing with low depreciation rates has high stockflow ratios so even relatively large changes in housing production over a moderate time period have small effects on the total stock, causing only minor changes in the service flows. Second, if \( \delta_h \) is sufficiently low, the shadow value will be mainly affected by the marginal utilities of service flows in the distant future. Since the effects of the shock are temporary, the future terms in this equation are close to their steady-state values. Thus, even if there were significant changes in the first few terms of the expansion, they would have a small percentage effect on the present value. As a consequence, under the benchmark, demand for durable housing displays an almost infinite elasticity of intertemporal substitution and the demand curve at any given time is very flat.

\(^{13}\)All other conditions remain the same as before. The introduction of asset adjustment cost is to induce stationary dynamics in a small open frictionless economy but it does not affect the quantitative results of the model since \( \phi_d \) is very small. For more details, see Schmitt-Grohe and Uribe (2003)
3.5 Calibration

The model period is a quarter. Preference: Following Schmitt-Grohe and Uribe (2003), the inverse of elasticity of substitution in consumption, $\sigma$, and the elasticity of labor supply, $\omega$, are set to 2 and 1.6, respectively. The elasticity of substitution between non-durable goods and housing service, $\eta$, is set to unity. The parameter $\gamma$ is set so that the ratio of private residential investment over GDP is equal to 3.5%, the average level for the U.K private residential investment over recent 20 years. Discount factor $\beta$ is chosen as 0.985, which is a bit lower than the value implied by the foreign real interest rate $1 + 0.01 = 0.9901$ to assure the binding credit constraint at steady state. The parameter $\kappa$ is selected so that a fraction $\frac{14}{24 \times 7}$ of household’s one unit time endowment is used for working in the labor market.

Technology: The share of capital in the production of non-durables and housing, $\alpha_c, \alpha_h$, are both set to 0.3. These parameters and depreciation rates will determine the non-housing investment rate, which is 20% of GDP. The depreciation rate of non-housing capital is chosen at 12% per year or $\delta_k = 0.03$ whereas $\delta_h$ is set to 0.003 or the depreciation rate of housing is 1.2% annually. Capital adjustment cost parameter $\phi_k$ is chosen such that non-housing investment volatility in this small open economy model matches the data.

Steady state value of the real world interest rate is set at 4% per year or $r=0.01$. The persistence coefficient $\rho_A$ in the motion equation of the productivity shock $\log(A_{t+1}) = \rho_A \log(A_t) + \epsilon_{t+1}$ is set to 0.9 and the variance of the innovation is selected to match the volatility of output.

For U.K: I set borrowing constraint parameter $\phi$ to 0.4 compared to 0.6 of the U.S economy. The reason for assigning 0.6 to the U.S economy is as follows. First, for the first-time home-buyers, the down-payment rate is typically less than 20%, which means these households can borrow more than 80% of the housing collateral value. For existing homeowners, Mian and Sufi (2009) show that these households on average extract 30 cents for every dollar increase in home equity. I take average of these numbers, which implies a value of 0.6 for the US. Then I scale down the IMF mortgage market development index so

\[ \frac{1}{1+0.01} = 0.9901 \]

There is no consensus about this. Piazzesi et al. (2007) argue that if $\eta$ is sufficiently less than unity then the equity premium puzzle is resolved while Davis and Martin (2005) show that the value should be no less than 1.25 in order to be consistent with U.S housing stock and price data. This paper take a neutral stance to set the value to unity. Similar qualitative results are obtained for those assigned in the neighborhood of unity.

Monacelli sets it to 1% while Davis and Heathcote and others use 1.56% per year.
that 0.98 is equal to 0.6 and obtain the number 0.4 for the U.K accordingly. At the same time, standard deviation of technology innovation and capital adjustment cost parameter $\phi_k$ are calibrated to match the standard deviation of output and non-housing investment in the U.K over the past 30 years, 1.15% and 4.10% respectively.

### 3.6 Housing Investment Dynamics

I first fix $\phi$ and explore the model’s responses to a positive productivity shock with an emphasis on the dynamics of housing investment. A favored productivity shock reduces costs so encourages firms in both durable and non-durable sectors to hire more labor and extend production. As a result, the housing supply curve shifts down to the right.

From the demand side, aggregate consumption increases because of the income effect. Since housing is a normal good, its demand goes up, leading to an upward shift in the demand curve, which applies for both free borrowing and credit constraint cases. The differences, however, lie on the combination of the income effect and potentially, the financial accelerator effect of each demand’s structure.

Figure 11 presents impulse responses of the free borrowing model. In a free borrowing environment, housing is just a durable consumption good, therefore, increases in housing demand come only from the income effect. Consequently, while these increases in housing demand cause real housing prices and housing investment positively co-move with non-housing investment and output, it is shown in Table 3 that is not sufficient to generate a realistic response/volatility of housing investment.

![Figure 11 about here.]

![Figure 12 about here.]

By contrast, Figure 12 presents impulse responses of the housing collateral constraint model. There are notably two main differences from the demand side of housing. First, unlike households that smooth consumption over time in the free borrowing benchmark, borrowing-constrained households are impatient, hence tend to locate consumption toward the current period. Second, since housing plays an additional role as a collateralizable asset and increased collateral value will enable credit constrained households to

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\[^{16}\text{This implies that correlated productivity shocks in the two sectors, combined with an exogenous interest rate in the small open economy, can produce the correct co-movement of housing investment and real housing prices in this type of two sector model, which is consistent with Greenwood and Hercowitz (1991), and Baxter (1996).}\]
expand consumption by further borrowing, households will have extra incentives to invest in housing. In other words, besides the direct utility gain from housing services, households will also obtain the benefit of relaxing the borrowing constraint with additional collateralizable housing assets. This is shown in the RHS of the demand equation (3.10): In addition to the standard discounted stream of utility from housing services, there is the second term presenting the current and expected benefits from the opportunity to expand consumption thanks to increased collateral value. The collateral constraint creates a borrowing-consuming spiral or a financial accelerator: initial increases in housing demand raise housing prices and stock so increase the collateral value, enabling credit-constrained households to borrow more for consumption, which in turn reinforce the demand for housing. As shown in Figure 13, housing investment in the borrowing constraint model is significantly more responsive and persistent than that of the benchmark and Table 3 shows the financial accelerator can account for the highly volatile behavior of housing investment. In particular, the calibrated basic model with a housing collateral constraint explain about 90% of housing investment volatility in the UK economy whereas the benchmark model of free borrowing can only explain about 47% of the volatility.

[Figure 13 about here.]

[Table 3 about here.]

### 3.7 Comparative Analysis of Mortgage Market Liberalization

This subsection examines the impact of mortgage market liberalization on housing investment by imposing different values of parameter $\phi$ in the borrowing constrained economy. In more liberalized mortgage markets (higher $\phi$), households can borrow a higher amount of debt for the same value of collateral and withdraw more equity from increased collateral value for consumption. More liberalized mortgage markets, therefore, intensify the collateral role of housing and this is the so-called collateral effect of mortgage market liberalization and innovation. The collateral effect leverages and encourages credit constrained households to invest more houses in good times. A higher $\phi$ also strengthens the link between the housing market and consumption decisions, therefore creating a relatively stronger financial accelerator.

On the other hand, mortgage market liberalization, by raising $\phi$, also offers the prospect of increased credit supply and a relaxation of borrowing constraints, therefore

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17The ratio of private residential investment over GDP is re-calibrated to 5%, which is the average ratio across these 17 OECD countries.
creating the so-called credit effect for the economy. In contrast to the collateral effect, the credit effect by providing more credit supply to the economy reduces the shadow value of borrowing credit, which in turn mitigates the incentive to invest in collateralizable housing assets. This credit effect was particularly emphasized before the onset of the financial meltdown 2007/8.

The collateral effect and the credit effect are partly reflected through the second term in the housing demand equation (3.10): A higher value of $\phi$ directly increases the value of this term but at the same time eases the tightness of the borrowing constraint, hence decreasing the current and future shadow value of borrowing $\lambda_{t+j}$. Moreover, a higher $\phi$ also leads to changes in the housing demand, hence affecting future expected housing prices $q_{t+1+j}$, which then in turn have impacts on the second term of RHS of equation (3.10) as well. It turns out that at low and medium levels of mortgage market development and liberalization, when the household’s credit constraint is relatively tight, the collateral effect prevails. Consequently, an increase in mortgage market liberalization leads to a relatively more responsive housing demand, leading to a higher housing investment volatility. By contrast, when the mortgage market is highly liberalized, households are much less credit constrained, so the credit effect takes over from the collateral effect, and the household starts to substitute collateralizable housing by non-durable consumption; therefore, housing investment volatility tends to decline. Figure 14 presents an inverse U-shape in the relative volatility of housing investment with respect to the degree of mortgage market liberalization.

[Figure 14 about here.]

[Figure 15 about here.]

4 Extended Model

The basic model with a representative agent is straightforward and sufficient to explain the empirical findings and the the financial accelerator of the housing collateral constraint. However, it has some limitations. First, it could not account for households’ heterogeneity so ignore the fact that not all households are borrowing constrained and use their housing as “ATM machine”. A representative agent model may therefore over-state the financial accelerator. Regarding to heterogeneity in wealth distribution, the top 20% (25%) of the wealth distribution in the US (UK) account for 82.6% (75%) of total wealth in the economies (Davies et al., 2008). In particular, for the US, Diaz and Luengo-Prodo (IER
forthcoming) document that the top 20% of the wealth distribution in the US own almost all (98.9%) total financial assets. Housing wealth accounts for 96.3% of total wealth for the bottom 80% but only for 26.8% for the top 20% of the wealth distribution. For the U.K, the wealthiest 20% owned 84% of net financial wealth (Wealth in Great Britain: 2006/8 Report).

Second, there is an anomaly between the basic model and data. That is the volatility of non-housing investment also increases in economies with more liberalized mortgage markets (Figure 15). The reason is that in the basic model, the only one representative agent also holds non-housing capital and the borrowing constraint makes the return rates of non-housing capital higher than the exogenous borrowing rates. Hence, increased credit supply from mortgage market liberalization (the credit effect) will allow and encourage the representative agent to invest more in non-housing capital. However, in reality only a fraction of population, who are wealthy enough, own/invest in non-housing (regular business) capital. The extended model is going to take into account the household heterogeneity and to correct the anomaly.

In particular, in the extended model, I assume a fraction, \( \epsilon \), of the population have a high discount factor so be patient enough to eventually become savers. These savers hold all non-housing capital so are called the *capitalists*. The other fraction, \( 1 - \epsilon \), of the population in the bottom of the wealth distribution are impatient so become borrowers. Housing is borrowers’ sole asset and is used as collateral. Consequently, housing heterogeneity is also aimed to address a feature of this paper that housing is residential properties, excluded commercial estates and only housing stock/capital is used as collateral in households’ borrowing.

Finally, I also assume that savers/capitalists have access to both international and domestic financial markets whereas borrowers only can borrow from the domestic bond market.

### 4.1 Capitalist

The representative capitalist maximizes his expected life-time utility defined over random sequences of non-durable consumption goods, \( c_{1t} \), housing services from housing stock, \( h_{1t} \), and labor dis-utility, \( l_{1t} \):

\[
U = E_0 \sum_{t=0}^{\infty} \beta^t U(c_{1t}, h_{1t}, l_{1t})
\]  

(4.27)
The budget constraint of the capitalist is given by:

\[
c_{1t} + q_t[h_{1t} - (1 - \delta_h)h_{1t-1}] + \bar{c}_t + \bar{r}_t + (1 + r_{t-1})d_{ft-1} + (1 + r_{t-1}^d)d_{lt-1} + \phi_d(d_{ft} - \bar{d})^2 \leq w_t l_{1t} + r^c_t k^c_{t-1} + r^h_t k^h_{t-1} + d_{lt} + d_{ft} \tag{4.28}
\]

\[
i_t^c = k_t^c - (1 - \delta_k)k_{t-1}^c + \phi_k (k_t^c - k_{t-1}^c)^2 \quad \frac{2}{k_{t-1}^c}
\]

\[
i_t^h = k_t^h - (1 - \delta_k)k_{t-1}^h + \phi_k (k_t^h - k_{t-1}^h)^2 \quad \frac{2}{k_{t-1}^h}
\]

Each period, the capitalist can either pay adjustment cost, \(\frac{\phi_d}{2}(d_{ft} - \bar{d})^2\), to borrow internationally traded foreign debt at an exogenous interest rate, \(r\), or access the domestic bond market, \(d_{lt}\), at an interest rate, \(r^d\). He supplies labor, \(l_{1t}\), at the real wage rate, \(w_t\), and lends capital, \(k_{t-1}^c, k_{t-1}^h\), to capital markets at prices, \(r^c, r^h\). The capitalist then spreads his income on non-durable tradable consumption goods, \(c_{1t}\), debt payment \((1 + r_{t-1})d_{ft-1}, (1 + r^d_{t-1})d_{lt-1}\), investments of two types of non-housing capital, \(i_t^c, i_t^h\), and housing investment, \(q_t[h_{1t} - (1 - \delta_h)h_{1t-1}]\).

The first order conditions for the capitalist, which are standard, read:

\[
U_{1ct}[1 + \phi_k (\frac{k_t^c - k_{t-1}^c}{k_{t-1}^c})] = \beta_1 E_t\{U_{1ct+1}[1 - \delta_k + r^c_{t+1} + \frac{\phi_k}{2}((\frac{k_{t+1}^c}{k_t^c})^2 - 1)] \}
\]

\[
U_{1ct}[1 + \phi_k (\frac{k_t^h - k_{t-1}^h}{k_{t-1}^h})] = \beta_1 E_t\{U_{1ct+1}[1 - \delta_k + r^h_{t+1} + \frac{\phi_k}{2}((\frac{k_{t+1}^h}{k_t^h})^2 - 1)] \}
\]

\[
w_t = -\frac{U_{1lt}}{U_{1ct}}
\]

\[
U_{1ct}(1 - \phi_d(d_{ft} - \bar{d})) = \beta_1 E_t\{U_{1ct+1}(1 + r_t) \}
\]

\[
U_{1ct} = \beta_1 E_t\{U_{1ct+1}(1 + r^d_t) \}
\]

\[
q_t U_{1ct} = U_{1ht} + \beta E_t\{U_{1ct+1}[q_{t+1}(1 - \delta_h)]\}
\]

4.2 Borrower

The representative borrower maximizes his expected life-time utility defined over random sequences of non-durable consumption goods, \(c_{2t}\), housing services from housing stock, \(h_{2t}\), and labor dis-utility, \(l_{2t}\):

\[
U = E_0 \sum_{t=0}^{\infty} \beta_t U(c_{2t}, h_{2t}, l_{2t}) \tag{4.32}
\]

\(\text{The introduction of adjustment costs in a small open economy framework is to induce stationarity.}\)
The budget constraint of the borrower is given by:

\[ c_{2t} + q_t[h_{2t} - (1 - \delta_h)h_{2t-1}] + (1 + r^d_{t-1})d_{2t-1} \leq w_t l_{2t} + d_{2t} \] (4.33)

I assume that the borrower is more impatient than the capitalist or \( \beta_2 < \beta_1 \). The borrower does not hold capital. \(^{19}\) Each period, he supplies labor, \( l_{2t} \), at the real wage rate, \( w_t \), borrows from the domestic bond market, \( d_{2t} \), at the interest rate, \( r^d_t \). The borrower then spreads his income on non-durable tradable consumption goods, \( c_{2t} \), debt payment, \((1 + r^d_t)d_{2t-1}\), and housing investment, \( q_t[h_{2t} - (1 - \delta_h)h_{2t-1}] \).

The borrower is also subject to the following collateral borrowing constraint:

\[ (1 + r^d_t)d_{2t} \leq \phi E_t(q_{t+1}h_{2t}) \] (4.34)

Denote the multiplier on the borrowing constraint by, \( \lambda_t \), then the first-order conditions for the borrower read:

\[ w_t = -\frac{U_{2t}}{U_{2c}t} \] (4.35)

\[ U_{2c}t - \lambda_t = \beta_2 E_t\{U_{2c+1}(1 + r^d_t)\} \] (4.36)

\[ q_t U_{2c}t = U_{2ht} + \lambda_t \phi E_t(q_{t+1}) + \beta E_t\{U_{2c+1}[q_{t+1}(1 - \delta_h)]\} \] (4.37)

### 4.3 Firms

Tradable and non-durable goods are produced by the following technology:

\[ y_t = A_t(k_{t-1}^c(l_t^c)^{1-\alpha_c}) \] (4.38)

Structures of the non-tradable durable housing are produced with following technology:

\[ b_t = A_t(k_{t-1}^h(l_t^h)^{1-\alpha_h}) \] (4.39)

Optimality conditions of non-durable goods firms imply:

\[ w_t = (1 - \alpha_c)\frac{y_t}{l_t^c} = (1 - \alpha_c)A_t\left(\frac{k_{t-1}^c}{l_t^c}\right)^{\alpha_c} \] (4.40)

\[ r^c_t = \alpha_c\frac{y_t}{k_{t-1}^c} = \alpha_cA_t\left(\frac{k_{t-1}^c}{l_t^c}\right)^{\alpha_c-1} \] (4.41)

Optimality conditions of construction firms imply:

\[ w_t = q_t(1 - \alpha_h)\frac{b_t}{l_t^h} = q_t(1 - \alpha_h)A_t\left(\frac{k_{t-1}^h}{l_t^h}\right)^{\alpha_h} \] (4.42)

\[ r^h_t = q_t\alpha_h\frac{b_t}{k_{t-1}^h} = q_t\alpha_hA_t\left(\frac{k_{t-1}^h}{l_t^h}\right)^{\alpha_h-1} \] (4.43)

\(^{19}\)It can be shown that because of being relatively impatient, the borrower will not hold capital.
4.4 Equilibrium

Given the interest rate, $r$, a competitive equilibrium in this small open economy is characterized by a sequence of allocations $\{c_{1t}, c_{2t}, l_{1t}, l_{2t}, h_{1t}, h_{2t}, d_{1t}, d_{2t}, d_{ft}, k_{1t}, k_{2t}, i_{1t}, i_{2t}, y_{t}, l_{ct}, l_{ht}, l_{dt}, k_{ct}, k_{ht}, i_{ct}, i_{ht}, y_{t}, l_{ct}, l_{ht}, l_{1t}, l_{2t}\}$, and a sequence of prices $\{q_{t}, w_{t}, r_{ct}, r_{ht}, r_{dt}, \lambda_{t}\}$ that satisfy the household and firms optimality conditions, the borrower’s budget constraint, the binding borrowing constraint, production functions, and following market clearing conditions.

Labor market clearing:
\[ \epsilon l_{1t} + (1 - \epsilon)l_{2t} = l_{ct} + l_{ht} \quad (4.44) \]

Non-tradable durable housing market clearing:
\[ b_{t} = \epsilon (h_{1t} - (1 - \delta_{h})h_{1t-1}) + (1 - \epsilon)(h_{2t} - (1 - \delta_{h})h_{2t-1}) \quad (4.45) \]

Domestic bond market:
\[ \epsilon d_{1t} + (1 - \epsilon)d_{2t} = 0 \quad (4.46) \]

 Tradable goods market:
\[ \epsilon c_{1t} + (1 - \epsilon)c_{2t} + i_{1t} + i_{2t} + (1 + r_{t-1})d_{ft-1} = y_{t} + d_{ft} \quad (4.47) \]

4.5 Calibration

Preference: Parameters such as $\sigma$, $\omega$, $\eta$, are chosen as those in the basic model. Capitalists and borrowers have the same share of housing services in the composition consumption and $\gamma_{1}, \gamma_{2}$, are set so that the aggregate housing investment GDP ratio is equal to 3.5%. Capitalists’ discount factor, $\beta_{1}$, is pinned down the exogenous world interest rate, $\frac{1}{1+r}$. Borrowers are more impatient so, $\beta_{2}$, is set to be 0.985 as in the basic model. $\kappa_{1}$, $\kappa_{2}$ is selected so that in steady state both capitalists and borrowers supply a fraction, $\frac{40}{247}$, of household’s total one unit time endowment for working in the market. I set the fraction of capitalists in total population, $\epsilon$, equal to 0.2, which implies that the top 20% of the wealth distribution in the model economy own regular business capital. The calibration is to match the fact that the top 20% of the wealth distribution holds most of total financial wealth.

Technology: All parameters related to the technology and productivity side of the model are kept the same as those in the basic model.

I calibrate the model such that steady state trade-balance-to-GDP ratio is equal to 1%, which then pins down the level of foreign debt at steady state $\bar{d}$.\(^{20}\) I also follow Schmitt-Grohe et al. (2003) to set the portfolio adjustment cost $\phi_{d}$ to 0.0007.

\(^{20}\)Although varying this ratio does not have much effect on our results.
4.6 Model Dynamics

When a favored productivity shock hits the economy, firms in both sectors hire more labor to extend production, driving up the wage rate and capital returns. Because of the income effect, both capitalists and borrowers increase their aggregate consumption, hence raising non-durable consumption. However, there is a contrast in the housing demand among capitalists and borrowers. For capitalists, housing is just a durable consumption good so as explained by Barsky et al. (2007), their elasticity of intertemporal substitution for long-lived housing is almost infinite so even a small rise in price relative to future will lead them to delay current purchase. Since the relative housing prices goes up on impact of the positive shock, capitalists optimally substitutes their durable consumption with non-durable goods, hence reducing their housing stock in the early stage and then gradually accumulates it back later on. By contrast, to the borrower, housing is not just a durable good but is also used as collateral, which therefore makes him increasingly invest in housing in good times. As shown in the simulation, the increase in the borrower’s housing demand not only is able to absorb the sale out of the capitalist’s housing but also drives up the aggregate housing investment (Figure 16).

Moreover, since non-housing capital is entirely owned by capitalists who are assumed not to be subject to borrowing constraints, its dynamics are not affected by the liberalization of the mortgage market. As a result, unlike housing investment whose volatility relative to GDP increases in economies with more liberalized mortgage markets, the volatility of non-housing investment remains almost unchanged, which is consistent with the empirical evidence (Figure 18).

Finally, I calibrate the extended model for the U.K and Table 4 presents the result. In particular, the calibrated extended model can explain 80% of the volatility of housing investment for the UK. Moreover, unlike the benchmark, the credit constraint models also can account for the significant counter-cyclicality of the current account. That is because households are credit-constrained they tend to borrow more (from foreigners) to consume in good times and the financial accelerator reinforces borrowing. However, the model has its limit: the implied volatility of housing prices of credit constraint models,
which is although about 2-3 times higher than that in the free borrowing model, is far below that of the data. This reflects the difficulty of business cycle models in accounting for the high volatility of asset prices.

[Table 4 about here.]

5 Conclusions

This paper begins by documenting stylized facts regarding housing investment and mortgage market innovation and liberalization in OECD countries. Housing investment is highly volatile, especially in economies with more liberalized mortgage markets. The paper demonstrates that standard DSGE models with a perfect credit market assumption are at odds with these empirical facts but the introduction of a housing collateral constraint can help reconcile the models with the facts. Collateral effects also enable the models to produce significant counter-cyclicality of the current account and the co-movement of different types of investments even without highly correlated productivity shocks.

Appendix: Data

**House Prices**: from (1) File Residential\_property\_prices.csv from the BIS via Markus Kramer, (2) Residential Prop prices IT.xls for Italy from Nomisma. Japanese house prices, however, are taken from Datastream with Code name JPLANDPIF.


**Housing investment, non-residential investment, aggregate investment**: real values, SA. Sources: Datastream, OECD Stats. Code means Datastream Code.


References


### Table 1: Statistic I

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<td>4.43</td>
<td>4.29</td>
<td></td>
</tr>
<tr>
<td>Norway</td>
<td>0.49</td>
<td>0.23</td>
<td>-0.08</td>
<td>-0.01</td>
<td>4.85</td>
<td>5.08</td>
<td>4.31</td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>0.33</td>
<td>0.11</td>
<td>0.51</td>
<td>0.54</td>
<td>4.50</td>
<td>4.33</td>
<td>3.33</td>
<td></td>
</tr>
<tr>
<td>Sweden</td>
<td>0.77</td>
<td>-0.44</td>
<td>0.78</td>
<td>0.60</td>
<td>6.15</td>
<td>2.85</td>
<td>2.23</td>
<td></td>
</tr>
<tr>
<td>U.K</td>
<td>0.58</td>
<td>0.57</td>
<td>0.48</td>
<td>0.72</td>
<td>6.44</td>
<td>3.60</td>
<td>3.16</td>
<td></td>
</tr>
<tr>
<td>U.S</td>
<td>0.40</td>
<td>0.64</td>
<td>0.78</td>
<td>0.91</td>
<td>6.67</td>
<td>3.57</td>
<td>3.40</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.47</td>
<td>0.42</td>
<td>0.64</td>
<td>0.72</td>
<td>4.63</td>
<td>3.77</td>
<td>3.24</td>
<td></td>
</tr>
</tbody>
</table>

Notes: HP denotes real house prices, RES is real housing investment, NRES is real non-housing investment, INV is real aggregate investment, GDP is real GDP. Correlations are correlation with GDP, RES/GDP, NRES/GDP, and INV/GDP denote the relative volatility of RES, NRES, and INV to that of real GDP, respectively. All series are in logs and Hodrick-Prescott filtered.

### Table 2: Statistic II

<table>
<thead>
<tr>
<th>Country</th>
<th>RES</th>
<th>GDP</th>
<th>RES/GDP</th>
<th>Prior 95</th>
<th>RES</th>
<th>GDP</th>
<th>RES/GDP</th>
<th>Post 95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>9.12</td>
<td>1.78</td>
<td>5.12</td>
<td>8.59</td>
<td>0.61</td>
<td>14.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Austria</td>
<td>3.99</td>
<td>0.78</td>
<td>5.12</td>
<td>2.51</td>
<td>0.80</td>
<td>3.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>7.63</td>
<td>1.94</td>
<td>3.93</td>
<td>4.19</td>
<td>0.87</td>
<td>4.82</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finland</td>
<td>5.70</td>
<td>2.50</td>
<td>2.28</td>
<td>6.00</td>
<td>1.05</td>
<td>5.71</td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>3.15</td>
<td>0.97</td>
<td>3.25</td>
<td>1.99</td>
<td>0.74</td>
<td>2.69</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>2.39</td>
<td>1.04</td>
<td>2.30</td>
<td>2.16</td>
<td>0.76</td>
<td>2.84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>New Zealand</td>
<td>8.07</td>
<td>1.77</td>
<td>4.56</td>
<td>8.40</td>
<td>1.05</td>
<td>8.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Norway</td>
<td>7.60</td>
<td>1.53</td>
<td>4.97</td>
<td>5.00</td>
<td>0.98</td>
<td>5.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K</td>
<td>9.67</td>
<td>1.51</td>
<td>6.40</td>
<td>3.90</td>
<td>0.38</td>
<td>10.26</td>
<td></td>
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</tr>
<tr>
<td>U.S</td>
<td>10.42</td>
<td>1.53</td>
<td>6.81</td>
<td>5.01</td>
<td>0.87</td>
<td>5.76</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>6.77</td>
<td>1.54</td>
<td>4.47</td>
<td>4.78</td>
<td>0.81</td>
<td>6.24</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: RES is real housing investment, GDP is real GDP. RES/GDP denotes the relative volatility of RES to that of real GDP. All series are in logs and Hodrick-Prescott filtered.
Table 3: **Statistics: Basic Model**

<table>
<thead>
<tr>
<th>Standard deviation</th>
<th>Data</th>
<th>Standard Model</th>
<th>Basic Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
<td>1.15</td>
<td>1.15</td>
<td>1.13</td>
</tr>
<tr>
<td>consumption</td>
<td>1.18</td>
<td>0.83</td>
<td>0.89</td>
</tr>
<tr>
<td>nres</td>
<td>4.1</td>
<td>4.14</td>
<td>2.32</td>
</tr>
<tr>
<td>tb/y</td>
<td>0.54</td>
<td>0.73</td>
<td>0.16</td>
</tr>
<tr>
<td>res</td>
<td>7.57</td>
<td>3.58</td>
<td>6.68</td>
</tr>
<tr>
<td>sd(res)/sd(y)</td>
<td>6.58</td>
<td>3.11</td>
<td>5.90</td>
</tr>
<tr>
<td>hp</td>
<td>5.13</td>
<td>0.08</td>
<td>0.11</td>
</tr>
</tbody>
</table>

**Correlation w/ output**

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Std. Model</th>
<th>Basic Model</th>
<th>Extended Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>consumption</td>
<td>0.73</td>
<td>0.94</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td>nres</td>
<td>0.48</td>
<td>0.55</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>tb/y</td>
<td>-0.31</td>
<td>-0.07</td>
<td>-0.63</td>
<td></td>
</tr>
<tr>
<td>hp</td>
<td>0.58</td>
<td>0.93</td>
<td>0.79</td>
<td></td>
</tr>
<tr>
<td>res</td>
<td>0.57</td>
<td>0.85</td>
<td>0.7</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Notes: Data is obtained from time series for the U.K from Q1-1981 to Q3-2007. Standard model is free borrowing model. Std() is standard deviation. nres: non-housing investment, tb/y: trade-balance output ratio, res: housing investment, hp: real housing prices. All numbers are in percentage, which is the standard deviations from trend and is obtained from Hodrick-Prescott filter.

Table 4: **Statistics: Extended Model**

<table>
<thead>
<tr>
<th>Standard deviation</th>
<th>Data</th>
<th>Std. Model</th>
<th>Basic Model</th>
<th>Extended Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
<td>1.15</td>
<td>1.15</td>
<td>1.13</td>
<td>1.16</td>
</tr>
<tr>
<td>consumption</td>
<td>1.18</td>
<td>0.83</td>
<td>0.89</td>
<td>0.87</td>
</tr>
<tr>
<td>nres</td>
<td>4.1</td>
<td>4.15</td>
<td>2.32</td>
<td>4.13</td>
</tr>
<tr>
<td>tb/y</td>
<td>0.54</td>
<td>0.73</td>
<td>0.16</td>
<td>0.69</td>
</tr>
<tr>
<td>res</td>
<td>7.57</td>
<td>3.58</td>
<td>6.68</td>
<td>6.21</td>
</tr>
<tr>
<td>sd(res)/sd(y)</td>
<td>6.58</td>
<td>3.11</td>
<td>5.90</td>
<td>5.35</td>
</tr>
<tr>
<td>hp</td>
<td>5.13</td>
<td>0.08</td>
<td>0.11</td>
<td>0.17</td>
</tr>
</tbody>
</table>

**Correlation w/ output**

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Std. Model</th>
<th>Basic Model</th>
<th>Extended Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>consumption</td>
<td>0.73</td>
<td>0.94</td>
<td>0.98</td>
<td>0.97</td>
</tr>
<tr>
<td>nres</td>
<td>0.48</td>
<td>0.55</td>
<td>0.85</td>
<td>0.53</td>
</tr>
<tr>
<td>tb/y</td>
<td>-0.31</td>
<td>-0.07</td>
<td>-0.63</td>
<td>-0.2</td>
</tr>
<tr>
<td>hp</td>
<td>0.58</td>
<td>0.93</td>
<td>0.79</td>
<td>0.9</td>
</tr>
<tr>
<td>res</td>
<td>0.57</td>
<td>0.85</td>
<td>0.7</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Notes: Data is obtained from time series for the U.K from Q1-1981 to Q3-2007. Extended model denotes the heterogeneous household model with credit constraint, standard model is the benchmark of free borrowing. Std() means standard deviation. nres: non-housing investment, tb/y: trade-balance output ratio, res: housing investment, hp: real housing prices. All numbers are in percentage, which is the standard deviations from trend and is obtained from Hodrick-Prescott filter.
Figure 4: Mortgage Depth Development

Figure 5: GDP Volatility and MMI

\[ \text{Corr:0.47 (p-value:0.06), } R^2=0.22 \]

Figure 6: GDP Volatility and MD

\[ \text{Corr:0.22 (p-value:0.4), } R^2=0.07 \]
Figure 7: Housing Investment Volatility and MMI

![Graph showing the relationship between Mortgage Market Index and Housing Investment Volatility to GDP. The correlation is 0.74 (p-value: 0.0) with an R^2 of 0.56.](image)

Figure 8: Housing Investment Volatility and MD

![Graph showing the relationship between Mortgage-debt-GDP-ratio and Housing Investment Volatility to GDP. The correlation is 0.6 (p-value: 0.01) with an R^2 of 0.4.](image)
Figure 9: Non-Housing Investment Volatility and MMI

Corr: 0.33 (p-value: 0.3) $R^2 = 0.11$

Volatility of Non-housing investment to GDP Fitted values

Figure 10: Non-Housing Investment Volatility and MD

Corr: 0.4 (p-value: 0.09) $R^2 = 0.21$

Volatility of Non-housing investment to GDP Fitted values
Figure 11: IRs to a Productivity Shock: Benchmark

Figure 12: IRs to a Productivity Shock: Borrowing Constraint
Figure 13: IRs of Housing Investment and Prices

Figure 14: Housing Investment Volatility and $\phi$

Figure 15: Non-Housing Investment Volatility and $\phi$