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#### Abstract

We consider the problem of accurate market risk modeling for agricultural commodity products over heterogeneous investment horizons using copulas and wavelet methods. Our results indicate that the degree and structure of the dependence of daily commodity returns on the three market risk factors (federal funds rate, USD/Euro exchange rate, and world stock market fluctuations) vary according to the time scale. Changes in the USD/EUR exchange rate and the stock market index are the dominant risks for agricultural commodity markets. Moreover, the tail dependence on the daily returns of the three market risk factors is also scale-dependent, and frequently asymmetric. Finally, there is evidence to suggest that the application of the wavelet-copula model improves the accuracy of VaR estimates, compared to traditional approaches.

JEL classification: Q14, C52, C58, G11, G17

*Keywords:* Agricultural commodities, Extreme-value copula, Wavelet, VaR, CVaR

#### I. Introduction

Fluctuations of prices are among the most important concerns for producers, consumers and investors acting in agricultural commodity markets, especially in agriculture-dominated countries. This has become a crucial matter over the last decade where market prices of agricultural commodities have experienced significant swings and extreme movements as witnessed by the recent evolution of the FAO Food Price Index (FFPI) in Figure 1.<sup>1</sup> The monthly deflated FFPI reached its first highest peak of 184.7 points in June 2008. representing respectively 36% and 73% higher than one year and four years earlier. The index has continued its upwards trend since February 2009 after a very short period of price decreases, and was 205.7 points in June 2011. The general surge in the price of all commodity groups is the main driver of the FFPI index. As an illustration, the dairy, oil & fat, and sugar price indices rose by more than 94% in real terms between February 2009 and June 2011. All in all, these price variations not only cause serious disruptions to international commodity markets, but also lead to higher costs for consumers and inflation threats as well.

It is commonly known that the price movements of agricultural commodities originate essentially from supply and demand conditions which, in turn, depend on weather patterns, seasonalities, market states, business cycles, and geopolitical situations (Lu, 2002; Giot and Laurent, 2003). Furthermore, while the climate change and natural disasters are largely beyond the control of market participants, also beyond their predictive abilities are the business cycle as well as the speculative behavior of some market participants. As a result of many such uncertainties, agricultural commodity prices can vary very substantially: thus hedging against their unfavorable movements through, for example, futures markets has become a primary and imperative task. The latter essentially requires an accurate modeling and forecasting of commodity price patterns.

In the previous literature various types of models have been used to exam-

<sup>&</sup>lt;sup>1</sup>The FFPI is a weighted average measure of the monthly change in international prices of a basket of five food commodity groups (meat, dairy, cereals, oil and sugar), where the weights are represented by the average export shares of each of the commodity groups for the period 2002-2004. There are in total 55 commodity quotations considered by FAO commodity specialists as representing the international prices of food commodities. Note that in February 2011, the FAO revised the composition of the meat price index, which consequently resulted in adjustments to the historical values of the FFPI.

ine dynamical changes in agricultural commodity prices. Tomek and Myers (1993) review a variety of frequently-used models that fall into the families of structural models and time-series models (e.g., linear structural modeling, stochastic trends analysis, cointegration, vector autoregressions and time-varying volatility models), and come to the conclusion that no single approach is the best, but research on commodity price analysis must have a clear statement of problems, an in-depth understanding of econometric models, and accuracy tests. Recent contributions to this topic almost all rely on time-series modeling techniques (Zanias, 1999; Chatrath et al., 2002; Giot and Laurent, 2003; Roache and Rossi, 2010; Gohin and Chantret, 2010; Nazlioglu, 2011; Natanelov et al., 2011). For example, Zanias (1999) tests the convergence of agricultural price series towards the law of one price by modifying the co-integration analysis to take seasonal components into account. Using data for the soft wheat market of five European Union member states, the author obtains mixed results because some markets turn out to be integrated whereas the integration hypothesis cannot be confirmed for the others. The study by Chatrath et al. (2002) shows that ARCH-type models, accommodating seasonality and contract-maturity effects, explain much of the nonlinearity contained in the futures prices of four important agricultural commodities. Giot and Laurent (2003) use ARCH-type VaR (Value-at-Risk) models to address the issue of market risk modeling in commodity markets including cocoa futures contracts, and find that the skewed Student APARCH model performs best in all cases. More recently, through the use of VECM and threshold cointegration, Natanelov et al. (2011) provide evidence that the co-movement between futures prices of crude oil, gold and agricultural commodities varies over time and some economic policies may change their nexus.

The above literature review shows that empirical models used in previous studies for modeling agricultural commodity price movements are exposed to a common pitfall in that they do not account for the potential outliers or irregular extreme values, which are likely to exist. The computation of VaR forecasts (market risk) reported in Giot and Laurent (2003) for example could, ultimately, be biased owing to the ignorance of the tail behavior of commodity-price distributions. Moreover, commodity prices may be characterized by multiscale structures, each occurring on a different time scale. This basic idea here is that the many types of market operators with heterogeneous risk preferences, capital budgeting constraints, information access, expectations, and risk perceptions may lead to their differential sensitivity to different time scales (e.g., hourly, daily, weekly or monthly). It then follows that commodity prices behave differently over different time scales, and there is a need to implement risk management strategies adapted to different time horizons.

In this paper we develop a wavelet-based copula framework to assess market risk for agricultural commodity products, related to fluctuations in the global stock, money, and foreign exchange markets. The combination of wavelet and extreme-value copulas allows us to explore not only the nature and the intensity, but also the asymmetry of dependence structure over different time scales. Our approach is also suitable for capturing the observed characteristics of commodity prices such as time-varying volatility and abrupt jumps (Deaton and Laroque, 1992; Myers, 1992), and skewed distributions and nonlinear price dynamics (Yang and Brorsen, 1992). In recent years, wavelet methods and copulas have separately received much attention from finance practitioners and researchers and they have been found to be useful in the study of the relationships between financial variables (see, e.g., Kim and In, 2005; Lada and Wilson, 2006; Durai and Bhaduri, 2009 for wavelet applications; and Jondeau and Rockinger, 2006; Chan and Kroese, 2010; and Aloui et al., 2011 for copula applications).

Using daily data for eight major agricultural commodities, federal funds effective rate, USD/EUR exchange rate, and MSCI world stock market index, we find that the degree and the structure of dependence of commodity returns with the three market risk factors vary according to time scales. Fluctuations in the USD/EUR exchange rate and the stock market index are the dominant risks for agricultural commodity markets. Market risks have negative effects on commodity returns and appear to be particularly high over the period from two to four business days (i.e., the shortest wavelet periodicity component) as indicated by the copula dependence parameters. For the longer periodicity component, this dependence parameter becomes positive for most cases, but smaller in size. Moreover, the interdependence during market extreme (positive or negative) movements is scale-dependent, and more often than not asymmetric. Our results also indicate that the proposed wavelet-copula model leads to a substantial improvement in the accuracy of VaR measures, as compared to traditional VaR estimation approaches.

The rest of this paper is organized as follows. Section 2 introduces the empirical framework and estimation procedure. Section 3 describes the data used and their statistical properties. Section 4 report the obtained results, while some concluding remarks are provided in Section 5.

# **II.** Empirical framework

In this section we introduce an integrated framework for examining the market risk of the main agricultural commodity products. First, the returns series are filtered using wavelet analysis. This method, recently applied to financial time series, decomposes a given returns series in orthogonal components, as in the Fourier approach, with respect to scales (time components) instead of frequencies. We then fit the suitable multivariate copula functions to different time components of various returns series in order to investigate their dependence structure at different time scales. We finally show how these results can be used to compute the Value-at-Risk (VaR) for agricultural commodity markets, conditionally on their dependence with changes in global equity market, US federal funds rate, and the dollar/euro exchange rate. In addition to some goodness-of-fit tests which aim to check the robustness of the results, we also distinguish between in-sample and out-of-sample periods so that we can compare the predictive power of our wavelet-based copula VaR with a standard VaR model.

#### Wavelet method for returns decomposition

Wavelet transform analysis has been found to be particularly useful for examining signals in both the time and frequency domains. Two statistical tools that are essential to our purpose include the maximal overlap discrete wavelet transform (MODWT) and multiresolution analysis (MRA).<sup>2</sup>

In wavelet analysis, any function f(t) in  $L^2(R)$  can be decomposed into components associated with different scales of resolution. More explicitly, the wavelet representation of the function f(t) is given by

$$f(t) = \sum_{k} S_{J,k} \phi_{J,k}(t) + \sum_{k} d_{J,k} \psi_{J,k}(t) + \sum_{k} d_{J-1,k} \psi_{J-1,k}(t) + \dots + \sum_{k} d_{1,k} \psi_{1,k}(t)$$
(1)

where  $\phi$  is the scaling function, also known as the father wavelet, and  $\psi$  is the wavelet function, also known as the mother wavelet,  $\phi_{j,k}$  and  $\psi_{j,k}$  are a scaling and translation of  $\phi$  and  $\psi$ , and  $S_{J,k}$  and  $d_{J,k}$ , respectively, are called the smooth coefficients and the detailed coefficients.

 $<sup>^{2}</sup>$ For a more complete review of the theory and the use of wavelets, see Percival and Walden (2000) and Gençay et al. (2002).

The MODWT filter is obtained directly from the discrete wavelet transform (DWT) filter. Let  $\phi_{j,k}$  and  $\psi_{j,k}$  denote the DWT scaling and wavelet filters, with k = 1, ..., K being the length of the filter and j the level of decomposition, the MODWT scaling  $\tilde{\phi}_{j,k}$  and wavelet  $\tilde{\psi}_{j,k}$  filters are given by

$$\widetilde{\phi}_{j,k} = \frac{\phi_{j,k}}{2^{j/2}} \text{ and } \widetilde{\psi}_{j,k} = \frac{\psi_{j,k}}{2^{j/2}}$$
 (2)

For a time series X with arbitrary sample size (N), the  $j^{th}$  level MODWT scaling  $(\widetilde{V}_i)$  and wavelet  $(\widetilde{W}_i)$  coefficients are defined by

$$\widetilde{V}_{j,t} = \sum_{k=0}^{K_j - 1} \widetilde{\phi}_{j,k} X_{t-k \mod N} \text{ and } \widetilde{W}_{j,t} = \sum_{k=0}^{K_j - 1} \widetilde{\psi}_{j,k} X_{t-k \mod N}$$
(3)

Note that the MODWT, which is a non-decimated form of the discrete wavelet transform (DWT), applies high-pass and low-pass filters to the input signal at each level. In contrast to the DWT, the output signal is not subsampled (not decimated) and the filters are upsampled at each level. On the other hand, while the DWT restricts the sample size N to an integer multiple of  $2^{j_0}$ , the MODWT is well defined for all sample sizes N. Furthermore, the MRA obtained with the MODWT is "shift invariant", i.e., shifting the time series by any amount will circularly shift each detailed and smooth coefficients by an equivalent amount. Overall, the decomposition of a time series into different components associated with different time scales would permit of obtaining a better description and understanding of the data generating process as well as the underlying dynamic market mechanisms.

#### Copula functions

Copulas are functions that link multivariate distributions to their univariate marginal functions. A good introduction to copula models and their fundamental properties can be found in Joe (1997) and Nelsen (1999). Formally, we refer to the following definition

**Definition 1.** A d-dimensional copula is a multivariate distribution function C with standard uniform marginal distributions.

Theorem 2. Sklar's theorem

Let  $X_1, ..., X_d$  be random variables with marginal distributions  $F_1, ..., F_d$ and joint distribution H, then there exists a copula  $C: [0, 1]^d \to [0, 1]$  such that

$$H(x_1, ..., x_d) = C(F_1(x_1), ..., F_d(x_d))$$
(4)

Conversely, if C is a copula and  $F_1, ..., F_d$  are distribution functions, then the function H defined above is a joint distribution with margins  $F_1, ..., F_d$ .

1

Another important concept used in this work is that of a survival copula. Given a copula C, the survival copula of C(u, v) is  $C_S(u, v) = u + v - 1 + C(1 - u, 1 - v)$ . The density of the survival copula is essentially a mirror image of the density of the original copula, i.e.,  $c_S(u, v) = c(1 - u, 1 - v)$ .

The copula models we consider here allow us to investigate both symmetric and asymmetric structures of dependence between variables.

- Normal copula: The bivariate normal (N) copula function is defined by

$$C(u,v) = \varphi_{\rho}(\varphi^{-1}(u),\varphi^{-1}(v)) \tag{5}$$

where  $\varphi_{\rho}$  is the standard bivariate normal distribution with linear correlation coefficient  $\rho$ , and  $\varphi$  represents the univariate standard normal distribution function. The normal copula can be rewritten as

$$C(u,v) = \int_{-\infty}^{\varphi^{-1}(u)\varphi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp(-\frac{s^2 - 2\rho st + t^2}{2(1-\rho^2)}) ds dt \qquad (6)$$
  
-1  $\leq \rho \leq 1, s = \varphi^{-1}(u), t = \varphi^{-1}(v)$ 

- Frank copula: The Frank (F) copula, which belongs to the Archimedean family, is given by

$$C(u,v) = -\frac{1}{\theta} \ln\left(1 + \frac{(\exp(-\theta u) - 1)(\exp(-\theta v) - 1)}{\exp(-\theta) - 1}\right), \ \theta \in (-\infty,\infty) \setminus \{0\}$$
(7)

- Clayton copula: The Clayton (C) copula is also an Archimedean copula and is given by

$$C(u,v) = (u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}, \ \theta \in [-1,\infty) \setminus \{0\}$$
(8)

- Survival Clayton copula: The Survival Clayton (SC) copula is derived from the Clayton copula

$$C(u,v) = u + v - 1 + ((1-u)^{-\theta} + (1-v)^{-\theta} - 1)^{-1/\theta}, \ \theta \in [-1,\infty) \setminus \{0\} \ (9)$$

- *Gumbel copula*: The Gumbel copula (G) is an extreme-value Archimedean copula, given by

$$C(u,v) = \exp\{-[(-\ln u)^{\theta} + (-\ln v)^{\theta}]^{1/\theta}\}, \ \theta \in (1,+\infty)$$
(10)

- Survival Gumbel copula: The Survival Gumbel (SG) copula is the mirror image of the Gumbel copula. It is given by

$$C(u,v) = u + v - 1 + \exp\{-[(-\ln(1-u))^{\theta} + (-\ln(1-v))^{\theta}]^{1/\theta}\}, \ \theta \in (1, +\infty)$$
(11)

- Tawn Copula: The Tawn (T) copula (or the mixed model of the Gumbel and independence copulas) is an extreme value copula given by

$$C(u,v) = uv \exp\{-\theta \frac{\ln u \ln v}{\ln(uv)}\}, \ 0 \le \theta \le 1$$
(12)

The elliptical copulas are the most popular in the finance literature due to their ease of use. The normal and the Student's t copulas fall into this family since they are based on an elliptically contoured distribution such as multivariate Gaussian or t distributions. The Gaussian copula is symmetric and has no tail dependence while the Student's t copula can capture extreme dependence between variables. For the normal, Student's t and Frank copulas,  $\theta = 0$  or  $\theta \to 0$  leads to independence, while  $\theta > 0$  and  $\theta < 0$  lead to positive and negative dependence, respectively.

Unlike elliptical copulas, the Archimedean copulas such as the Gumbel and Clayton copulas are not derived from multivariate distribution functions and can be used to capture asymmetry between lower and upper tail dependences. The Clayton copula exhibits greater dependence in the negative tail than in the positive, whereas the Gumbel copula exhibits greater dependence in the upper tail than in the lower tail. For the Clayton copula,  $\theta \to 0$  leads to independence, while  $\theta \to \infty$  leads to perfect positive dependence. For the Gumbel copula,  $\theta = 1$  and  $\theta \to \infty$  imply independence and perfect positive dependence, respectively. All Archimedean copulas are asymmetric, except for the Frank copula which enables capturing the full range of dependence for marginals exposed to weak tail dependence.

In order to fit copulas to our data, we use a semiparametric two-step estimation method, namely the Canonical Maximum Likelihood (Cherubini et al., 2007). This method first estimates the marginals  $F_X$  and  $G_Y$  non parametrically via their empirical cumulative distribution functions (ECDF)  $\hat{F}_X$  and  $\hat{G}_Y$ , defined by

$$\widehat{F}_X(x) = \frac{1}{n} \sum_{i=1}^n \mathbb{1}\{X_i < x\} \text{ and } \widehat{G}_Y(y) = \frac{1}{n} \sum_{i=1}^n \mathbb{1}\{Y_i < y\}$$
(13)

In the implementation,  $\widehat{F}_X$  and  $\widehat{G}_Y$  are rescaled by  $\frac{n}{n+1}$  to ensure that the first order condition of the log-likelihood function for the joint distribution is well defined for all finite n. Then the observations are transformed into uniform variates using the ECDF of each marginal distribution and the unknown parameter  $\theta$  of the copula is estimated by

$$\widehat{\theta}_{CML} = \arg\max_{\theta} \sum_{i=1}^{n} \ln c(\widehat{F}_X(x_i), \widehat{F}_Y(y_i); \theta)$$
(14)

Under suitable regularity conditions, the CML estimator  $\hat{\theta}_{CML}$  is consistent, asymptotically normal, and fully efficient at independence. Further details can be found in Genest et al. (1995). Moreover, before computing the ECDFs, we filter the returns with a standard GARCH(1,1) model to remove any serial dependence of the returns data as well as to capture stylized effects of their conditional volatility (i.e., persistence, clustering, and time-variations). After that, the CML estimation approach can be properly used.

To select the most appropriate copula model, we apply a goodness-of-fit test that investigates the distance between the estimated and the empirical copulas (Genest et al., 2009). The empirical copula  $C_n$ , which is the most famous and easiest nonparametric estimator for the copula of a random vector, basically represents an observed frequency and is obtained from the empirical margins. The distance between the said copulas is evaluated using a Cramér-von Mises statistic:

$$S_n = n \int \{C_n(u, v) - C_{\theta_n}(u, v)\}^2 dC_n(u, v)$$
(15)

Large values of  $S_n$  lead to the rejection of the null hypothesis that the estimated copula is closest to the empirical copula. In practice, we require knowledge about the limiting distribution of  $S_n$  which depends on the unknown parameter value  $\theta$ . To find the *p*-values associated with the test statistic we use a multiplier approach as described in Kojadinovic and Yan (2010).

## III. Data

Our sample data consist of daily closing spot prices for eight agricultural commodities widely traded in the Chicago Board of Trade (CBOT) and New York Board of Trade (NYBOT). The latter was renamed the Intercontinental Exchange (ICE) Futures US after September of 2007. Grain and oilseeds commodities include soft white winter wheat, live cattle, yellow soybeans, yellow corn, and soybean oil. Soft commodities include cocoa, cotton, and sugar. Commodity specifications are presented in Table 1. All data are expressed in US dollars, and obtained from Bloomberg and Reuters databases.

The three market risk factors considered in this paper are the federal funds effective rate (FFE.rate), the USD/EUR exchange rate (i.e., the amount of US dollars per Euro) and the world stock market index constructed by Morgan Stanley Capital International (MSCI). These variables, besides uncontrolled factors such as climate conditions and natural disasters, are of paramount importance for gauging the market risk faced by operators in agricultural markets since they directly affect production and investment decisions as well as international trade transactions. Collected daily, they are viewed as external shocks to agricultural price dynamics from changes in monetary policy, global trade, and stock market cycles.

The sample period is from October 3, 2003 to August 31, 2010, yielding 1789 observations. Returns series are computed by using the difference in

the logarithm of the two consecutive prices, except for the interest rate series whose changes are computed as the difference between two consecutive interest rates. Table 2 presents the summary statistics, while a graphical illustration of the commodity data and returns on market risk factors is shown in Figure 2 and Figure 3. Average daily returns for commodities range from 3% (Live cattle) to 7.2% (Sugar). Changes in federal funds rate are negative on average, which essentially reflects the decreasing trend of the funds rates during the course of the global financial crisis 2007-2009. We also observe that the US dollar appreciates against the euro over the study period, in view of the negative average returns on USD/EUR exchange rate. On the other hand, all commodity returns series exhibit high daily volatility in terms of standard deviations. All the series also appear to depart from normality as indicated by significant skewness and kurtosis coefficients. The Jarque-Bera statistics (J-B) confirm the fact that all the series are non-normally distributed. The Ljung-Box statistics for the 12-order autocorrelation are significant for six returns series. The results of the Ljung-Box test applied to the squared returns and the ARCH LM test give strong evidence of conditional heteroscedasticity in the returns series, which supports our decision to use a GARCH-type model to filter returns components obtained from wavelet decomposition. Finally, the results of common unit root tests (Dickey-Fuller and Phillips-Perron), not reported here to conserve space, show that all the series are stationary and thus suitable for further analysis.

#### IV. Results and discussions

## Time scales effects from wavelet decomposition

The wavelet approach described in the previous section is applied to decompose the returns on a scale-by-scale basis. Its use is motivated by the fact that commodity returns series seem to have both time and frequency domain representations, mainly due to the heterogeneity of market participants having different expectations, risk preference levels and investment horizons. For example, short-term investors are naturally more interested in short-term fluctuations of the prices, whereas long-term investors keep a close watch on long-run price movements. Typically, the evaluation of the time-varying comovement at different frequencies between commoditity returns and market risk factors is possible and very useful for investors to understand the changes in their market risk exposure over time across frequencies. Similar to previous studies in the related literature (Percival and Walden, 2000), we

	Description of agricultural commodities
Table 1	agricultural
	of
	Description

Series name Unit	Unit	Specifications	Sources	Bloomberg's ticker
Wheat	USD/bushel (60 lb)	USD/bushel (60 lb) USDA No. 1 bushel of soft white winter wheat (must test weigh at least 60 lb). Delivery by truck USDA averages 1,200 delivery, and by barge averages 50,000 bushels.	USDA	WEATPR1S Index
Cattle	USD/Hundredweigt	USDA cattle Live Slaughtered Steer prices are expressed in US dollars per hundredweight (45.36 USDA kg).	USDA	CATLLSPT Index
$\operatorname{Soybeans}$	USD/bushel (56 lb) USDA No. 1 l Illinois in 15 c	USDA No. 1 bushel of yellow soybeans must test weigh at least 56 lb. Bids for delivery in Chicago Illinois in 15 days.	USDA	SOYBCH15 Index
Corn	USD/bushel (56 lb) USDA Illinois bushel must te	USDA Illinois North Central No. 2 yellow-kerneled corn, which cannot contain more that 5grade USDA bushel must test weigh at least 54 lbs, delivered in Springfield, Illinois.	USDA	CORNILNC Index
Cotton	USD/pound	USDA South Delta strict-low middling is a medium quality cotton that contains fibers 1-1/16th USDA inches long. This price is a discount to the most active futures contract trading on the New York Cotton Exchange.	USDA	COTNSMSD Index
Soybean oil	USD/pound	USDA crude soybean oil spot price per pound is delivered in Springfield, Illinois.	USDA	SOYPIOIL Index
Cocoa	USD/bag (145 lb)	CSCE cocoa spot prices are expressed in US dollars per bag of 145 lb, delivered in New York.	USDA	COCAIVNY Index
Sugar	USD/pound	CSCE No. 11 sugar spot price index is a representative price for world sugar prices. The differential index is calculated by taking the difference between the sugar spot price and estimated average price on No. 11 price.	USDA	SUGARSPT Index

Notes: The table presents the specifications of daily spot price series for the eight agricultural commodities we study. Data are constructed by the United States Department of Agriculture (USDA) and extracted from Bloomberg and Reuters databases.

Panel A				
	Mean	Std Dev	Skewness	Excess kurtosis
Wheat	0.030	1.137	-0.389	8.334
Live cattle	0.003	1.456	0.121	5.281
Soybeans	0.022	1.841	-0.496	1.909
Cotton	0.018	1.947	-0.079	1.531
Soybean oil	0.018	1.816	0.052	1.819
Cocoa	0.024	4.236	3.733	47.651
Sugar	0.072	1.938	-0.116	1.360
Corn	0.036	2.089	-0.204	2.133
FFE. rate	-0.436e-03	0.102	-0.294	35.867
Euro/USD rate	-0.005	0.646	0.182	3.905
MSCI index	0.007	1.140	-0.455	10.376
Panel B				
	Q(12)	$Q^2(12)$	J-B	ARCH(12)
Wheat	207.907**	597.855**	5186.852**	$265.966^{**}$
Live cattle	138.994**	$278.086^{**}$	2068.003**	192.729**
Soybeans	17.106	432.173**	342.341**	184.778**
Cotton	10.337	413.807**	174.730**	185.771**
Soybean oil	14.273	409.485**	245.060**	187.801**
Cocoa	34.951	91.308**	172341.285**	69.308**
Sugar	23.094*	90.561**	140.451**	68.761**
Corn	16.249	409.795**	348.506**	172.120**
FFE. rate	275.235**	1615.917**	95311.86**	641.890**
Euro/USD rate	$19.077^{\dagger}$	307.088**	1137.702**	174.091**
MSCI index	48.520**	2383.582**	8032.425**	650.851**

Table 2	
Statistical properties of daily returns:	2003-2010

Notes: The table displays summary statistics for the returns data over the study period from October 3, 2003 through August 31, 2010. J-B, Q(12) and  $Q^2(12)$  are the empirical statistics of the Jarque-Bera test for normality and Ljung-Box test for serial correlation in returns and squared returns with 12 lags. ARCH(12) is the empirical statistics of the LM test for conditional heteroscedasticity applied to 12 lags. \*, and \*\* denote the rejection of the null hypothesis of no autocorrelation, normality and homoscedasticity at the 5% and 1% levels, respectively. <sup>†</sup> indicates the rejection of these null hypothesis at the 10% level.

use the Daubechies least asymmetric wavelet filter of level 8 (LA8) with periodic boundary conditions in the MODWT multiresolution decomposition. Each of the 11 time series was decomposed into four different periodicity series ranging from short to long-run periodicity: D1, D2, D3 and D4. These wavelet filter coefficients correspond respectively to 2-4, 4-8, 8-16 and 16-32 days period since we use daily data and set the number of scales J to be four. Additionally, the vector S4 captures the trend of the original returns series.

The results from the application of MODWT multiresolution decomposition to each time series, are presented in Figures 4 and 5. All the computations were performed using the S + Wavelets module running under the S - PLUS statistical computing environment. In each chart of Figures 4 and 5, we show the plot of the original series (sum), the scaling coefficient vector (S4) that captures the trend of the series, and the wavelet coefficient vectors from the small-scale component D1 (high frequency) to the high-scale component D4 (low frequency).

#### Wavelet variance and correlation analyses

Figures 6 and 7 show the MODWT-based wavelet variance estimates of the commodities and market risk returns as well as their corresponding 95% confidence intervals, represented by the upper (U) and lower (L) bounds. The associated graphs indicate that there is a general and common trend of decrease in the estimated wavelet variances as the scale increases. Investors in agricultural commodity markets with very short-term investment horizons are thus confronted with high risks. Commodities returns also exhibit significant differences in volatility over different scales. In particular, Cocoa returns experienced the highest level of volatility, while Wheat and Cattle have less volatile returns.

The estimated multiscale correlation coefficients between commodities returns and market risk factors are presented in Figures 8, 9 and 10. A very low degree of wavelet correlation is observed between commodities and the FFE rate at all scales considered. The expected effects on commodity prices of changes in policy interest rate are thus small. The wavelet correlation coefficients with the USD/EUR exchange rate and world stock market decrease significantly as the time scale increases. The most significant relationships are typically detected at the smallest scale, that is for the component D1. More precisely, the highest correlation, of about 0.30, is found between the MSCI world market index and Soybean oil, followed by the Corn-MSCI pair (0.27). Overall, the results suggest that commodities returns have higher correlations with the USD/EUR exchange rate and the MSCI world market index than with the FFE rate, and that the returns linkages are essentially stronger at the smallest scale.

## Dependence structure through copula functions

Copulas have recently become a very promising tool to assess the dependence between various time series in a flexible way. Combined with wavelets, they allow us to investigate both the strength, the structure and the timevariations of interdependence across different frequencies. Table 3 reports the results from the application of copula functions to each bivariate system of wavelet decomposed commodity returns and market risks. For each pair, the best copula model which is selected by the goodness-of-fit test (Genest et al., 2009) as well as its associated dependence parameter are presented. In Tables 4 and 5, we report the values of the upper and lower tail dependence coefficients, obtained from the best-fitting copula model. Two important findings emerge from the variety of patterns displayed in Tables 4 and 5: i) the degree and the structure of dependence are not constant across time scales; and ii) the interdependence during market extreme (positive or negative) movements is asymmetric and scale-dependent.

The results for the original series point to the survival Gumbel copula as the best-fitting model in five cases: Cattle-USD/EUR, Soybeans-USD/EUR, Cotton-MSCI, Sugar-USD/EUR and Sugar-MSCI. Notice here that the survival Gumbel copula highlights a strong relationship for the mentioned pairs, even for the negative values of the density function in the lower left corner. The Student's t copula gives the best fit for three pairs: Soybeans-MSCI, Cotton-USD/EUR and Soybean oil-USD/EUR. The Tawn copula is the best model only for Cattle-MSCI pair. For the remaining pairs, the fitted copula models are rejected at the 5% significance level. In particular, returns are positively linked whenever copula models are relevant. It turns out that rising commodity prices are typically associated with increases in the Fed interest rate (i.e., contractionary monetary policy), world stock market returns, and USD/EUR exchange rate (i.e., depreciation of the US dollar relative to the Euro). The tail dependence coefficients show that for a total of 24 pairs. we find extreme dependence during periods of large joint gains and losses for three pairs, extremely negative dependence for five pairs, and extremely positive dependence for only one pair (Tables 4 and 5). The Soybean oil-USD/EUR and the Soybeans-USD/EUR pairs show the highest degree of tail dependence during bull and bear markets, respectively.

Estimates of the copula and dependence parameters over five time scales Table 3

Notes: This table presents the copula parameter's estimates for each pair over different tima scales. The values between brackets represent the standard error of the parameters. \* Significance at the 1% levels, \*\* Significance at the 5% levels, \*\*\* Significance at the 10% levels.

Table 4Estimates of the upper tail dependence coefficients over five time scales

Pairs	C	D1	D2	D3	D4
	Sum	D1		20	D4
Wheat-FFE.rate	-	0	0	0	0
Wheat-USD/EUR	-	0.123	0.003	0	0
Wheat-MSCI	-	-	0.130	0.104	0
Cattle-FFE.rate	-	0	0	0	0
Cattle-USD/EUR	0	0	0	0	0.032
Cattle-MSCI	0.025	0	0	0	0
Soybeans-FFE.rate	-	0	0	0.085	-
Soybeans-USD/EUR	0	0	0	0.029	0
Soybeans-MSCI	0.052	-	0.042	0.017	0
Cotton-FFE.rate	-	0	0	0	0
Cotton-USD/EUR	0.018	0.115	0.033	0.010	-
Cotton-MSCI	0	0	0.062	0	-
Soybean oil-FFE.rate	-	0	0	0.120	-
Soybean oil-USD/EUR	0.087	0.065	0.029	0.033	0
Soybean oil-MSCI	-	-	-	-	-
Cocoa-FFE.rate	-	0	0	0	-
Cocoa-USD/EUR	-	0	0	0	-
Cocoa-MSCI	-	0	-	-	-
Sugar-FFE.rate	-	0	0	0	-
Sugar-USD/EUR	0	0	0	0.003	0.007
Sugar-MSCI	0	0.001	0.107	-	-
Corn-FFE.rate	-	0	0	0	_
Corn-USD/EUR	-	0.123	0	0	-
Corn-MSCI	-	-	0.034	0.004	0.053

Notes: This table presents the upper tail dependence parameter's estimates for each pair over different time scales.

Table 5Estimates of the lower tail dependence coefficients over five time scales

Dataa	0	D1	D0	D9	D4
Pairs	$\operatorname{Sum}$	D1	D2	D3	D4
Wheat-FFE.rate	-	0	0	0	0
Wheat-USD/EUR	-	0	0.003	0	0
Wheat-MSCI	-	-	0	0.104	0
Cattle-FFE.rate	-	0	0	0	0
Cattle-USD/EUR	0.022	0	0	0	0
Cattle-MSCI	0	0	0	0	0
Soybeans-FFE.rate	-	0	0	0	-
Soybeans-USD/EUR	0.162	0	0.137	0.029	0
Soybeans-MSCI	0.052	-	0.042	0.017	0
Cotton-FFE.rate	-	0	0	0	0
Cotton-USD/EUR	0.018	0	0.033	0.010	-
Cotton-MSCI	0.160	0.139	0.062	0.073	-
Soybean oil-FFE.rate	-	0	0	0	-
Soybean oil-USD/EUR	0.087	0.065	0.029	0.033	0
Soybean oil-MSCI	-	-	-	-	-
Cocoa-FFE.rate	-	0	0	0	-
Cocoa-USD/EUR	-	0	0	0	-
Cocoa-MSCI	-	0	-	-	-
Sugar-FFE.rate	-	0	0	0	-
Sugar-USD/EUR	0.094	0	0.082	0.003	0.007
Sugar-MSCI	0.096	0.001	0	-	-
Corn-FFE.rate	-	0	0	0	_
Corn-USD/EUR	-	0	0	0	-
Corn-MSCI	-	-	0.034	0.004	0.053

Notes: This table presents the lower tail dependence parameter's estimates for each pair over different time scales.

For the shortest periodicity component D1, fitted copula models are rejected only for four pairs. The class of Archimedean copulas is relevant in 12 out of the 20 remaining cases, while symmetric copulas (normal and Student's t) are selected as the best models in six cases. The Tawn copula is retained in two cases. These findings are consistent with the view that D1component commodity returns tend to be asymmetrically linked with market risk factors. We observe a negative relationship between the FFE rate and the following commodities: Wheat, Soybeans, Cotton, Soybean-oil, Cocoa, and Corn. The dependence parameter is also found to be negative for three other pairs: Cattle-USD/EUR, Cocoa-USD/EUR and Cocoa-MSCI. A close look at the tail dependence coefficients indicates that two pairs are mutually dependent in extremes on both positive and negative levels, three pairs during bull markets, and one pair during bear markets. The Cotton-MSCI pair has the strongest tail dependence during bear markets, while the Wheat-USD/EUR and Corn-USD/EUR pairs show the highest degree of dependence during bull market.

As to the medium periodicity components D2 and D3, we observe a net improvement of symmetric copulas in dependence modeling since they give the best fit in 11 and 13 cases, respectively. With respect to the component D2, the dependence is still negative for the following pairs: Corn-FFE.rate, Cotton-FFE.rate, Soybeans-FFE.rate, Soybean oil-FFE.rate, Wheat-FFE.rate, and Cocoa-USD/EUR. Other negative links are found and include Cattle-MSCI and Sugar-FFE.rate. The Wheat-MSCI and the Soybeans-USD/EUR pairs show the highest degree of dependence at extreme positive and negative levels, respectively. The D3-component dependence is positive for all pairs, except for Cocoa-FFE.rate, Cotton-FFE.rate, Wheat-FFE.rate, and Cattle-USD/EUR. The highest degree of tail dependence during periods of large joint losses and gains is found for the Wheat-MSCI and Soybean oil-FFE.rate pairs.

Twelve copula models are rejected at the 5% level for the longer periodicity component D4, and among the relevant copula models, the Student's tand normal copulas provide the best fit for four pairs. The dependence parameter is positive for all pairs, except for the Cattle-FFE.rate pair. A stronger tail dependence is observed for the pair Corn-MSCI at extreme negative and positive scenarios.

Overall, asymmetric copula functions appear to be best at modeling the dependence between commodity returns and market risk factors over the shortest and the longer periodicity components, while the role of Student's t

and normal copulas increases for medium periodicity components.

#### Wavelet-based copula models and accuracy of VaR estimation

Value-at-Risk is one of the most popular measures for market risk assessment. It is commonly defined by the maximum loss in a portfolio's value with a given probabiliy over a given time period. We now show how the proposed wavelet-copula approach can be used to improve the accuracy of VaR measurement. To this end, we compute the VaR of a representative equally-weighted portfolio, composed of the Soybeans and USD/EUR exchange rate. A backtesting procedure for the original returns series and for each periodicity components can be then implemented to assess the accuracy of the VaR estimates.

Methodologically, we proceed as follows. First, we estimate the whole model (i.e., copula-GARCH for raw series and wavelet-copula-GARCH for the decomposed series) using data only up to time  $t_0$ . We then simulate innovations from the copula and transform them into standardized residuals by inverting the marginal CDF of each series. Finally, we calculate the forecasting returns by using the GARCH-volatility and conditional mean terms, and compute the value of the considered portfolio. This procedure can be repeated until the last observation and we compare the estimated VaR with the actual next-day value change of the portfolio. The whole process is repeated only once in every 50 observations owing to the computational cost of this procedure and because we did not expect to see large modifications in the estimated model when only a fraction of the observations is modified. However, at each new observation the VaR estimates are modified because of changes in the GARCH volatility and the conditional mean.

We started by estimating the model using the first half of the data. Then, we simulate 3000 values of the standardized residuals, estimate the VaR and count the number of losses that exceeds the estimated VaR values. We also estimate the VaR using two other approaches: the variance–covariance (also known as analytical) and the historical simulation methods. While the first approach estimates the VaR assuming that the joint distribution of the portfolio returns is normal, the second measures the risk by means of ordered Loss–Profit observations. In the variance–covariance and historical simulation methods, the model parameters were updated for every observation. The results for the backtesting test are reported in Table 6. Note that a model is said to be best suited for calculating VaR is the one with the number of exceedances closest to the expected number of exceedances. We can

	results.
Table $6$	back testing
	VaR

Panel A									
Soybeans-USD/EUR		Sum			D1			D2	
σ	0.1(89)	0.05(45)	0.01(9)	0.1(89)	0.05(45)	0.01(9)	0.1(89)	0.05(45)	0.01(9)
Copula	0.091(81)	0.045(40)	0.010(9)	0.098(88)	0.054(48)	0.008(7)	0.098(88)	0.039(35)	0.011(10)
SH	0.120(107)	0.066(59)	0.018(16)	$0.127 \ (115)$	0.070(63)	0.017(15)	0.136(122)	0.066(59)	0.023(21)
NC	0.039(35)	0.017(15)	0.007(6)	0.035(31)	0.019(17)	0.002(2)	0.070(63)	0.041(37)	0.010(9)
Panel B									
Soybeans-USD/EUR		D3				D4	4		
σ	0.1(89)	0.05(45)	0.01(9)	0.1(89)	(89)	0.05	0.05(45)	0.01(9)	(6)
Copula	0.089 (80)	0.046(41)	0.006(5)	09) 2000	(09)	0.03	0.031(28)	0.002 (2)	2 (2)
SH	0.139(124)	0.072(64) $0.022(20)$	0.022(20)	0.131(117)	(117)	0.078	(078 (70)	0.023(21)	(21)
NC	0.101(90)	0.057(51) $0.017(15)$	0.017(15)	0.107(96)	(96)	0.06	0.066(59)	0.020(18)	(18)

Notes: This table reports the VaR backtesting results obtained from the GARCH-copula, historical simulation and variance covariance method over different time scales. see that wavelet-copula VaR models outperform both alternative approaches (analytical and historical simulation methods) for the original data and for each periodicity component at the conventional confidence levels. It provides less accurate VaR estimates only for the longest periodicity component D4 at the 90% and 95% confidence levels. Overall, these findings confirm the usefulness of wavelet-copula models in forecasting the market risk associated with investments in commodity markets.

#### V. Conclusion

Market risk forecasts for agricultural commodity prices are intended to be useful for farmers, governments, and agribusiness industries. They are indeed inputs for risk diversification and hedging strategies as well as regulation policies. In this paper, we propose a wavelet-copula framework to investigate the dependence structure of agricultural commodity prices with respect to three market risk factors: the federal funds effective rate, the USD/EUR exchange rate, and changes in the MSCI world stock market index. This framework allows us to capture not only the nature, but also the intensity of multiscale dependence structures as well as the possibility of asymmetric tail dependence. Applying this to eight major agricultural commodities, our results provide evidence that the intensity and structure of the dependence of commodity returns on the three market risk factors exhibit time-varying patterns over different time scales. Dynamic changes in commodity returns are the most sensitive to the movements of USD/EUR exchange rate and stock market index. The market risks have negative effects on commodity returns and appear to be particularly high over the period from two to four business days, but essentially positive effects for longer investment horizons. It then turns out that rising commodity prices are likely to be associated with increases in the Fed interest rate (i.e., contractionary monetary policy), world stock market returns, and USD/EUR exchange rate (i.e., depreciation of the US dollar relative to the Euro). Furthermore, the interdependence during market extreme (positive or negative) movements is scale-dependent, and more often than not asymmetric. We finally find that the proposed waveletcopula model leads to more accurate VaR forecasts than the traditional VaR approaches.

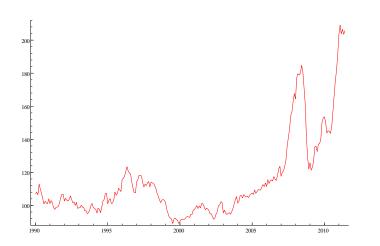
Overall, the findings presented in this paper offer insights into the timescale behavior of agricultural commodities in relation with the three major market risks. Adapting the investment strategies or regulation policies to time scale relationships is therefore desirable in order to avoid significant exposure to market risk.

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 $Figure \ 1. \ Dynamics \ of \ the \ monthly \ deflated \ FAO's \ food \ price \ index$ 

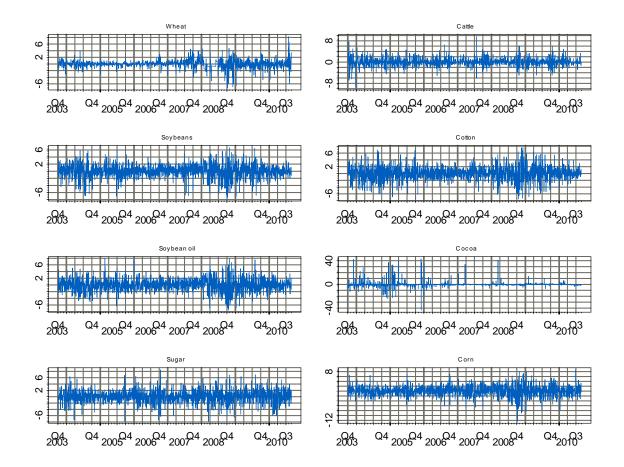


Figure 2. Time-variations of daily returns on agricultural commodities

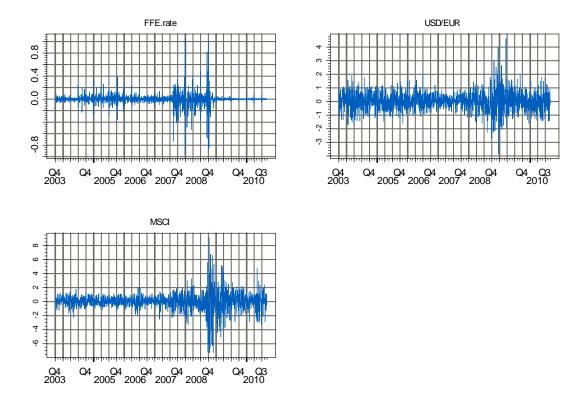


Figure 3. Time-variations of changes in federal funds effective rates and returns on USD/EUR exchange rate and MSCI world stock market index

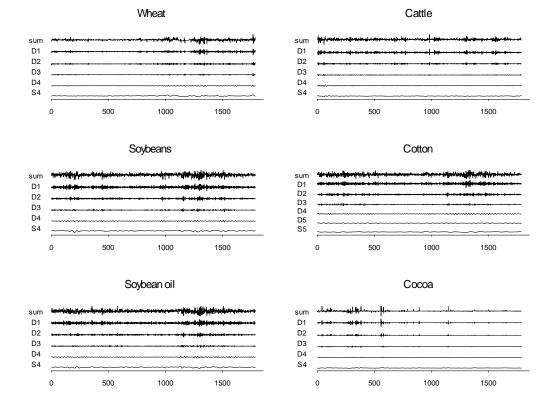


Figure 4. MODWT multiresolution decomposition of Wheat, Cattle, Soybeans, Cotton, Soybean oil and Cocoa returns series.

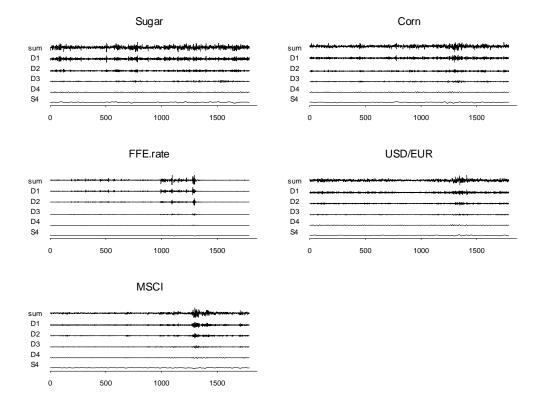


Figure 5. MODWT multiresolution decomposition of Sugar, Corn, FFE.rate, USD/Euro and MSCI returns series.

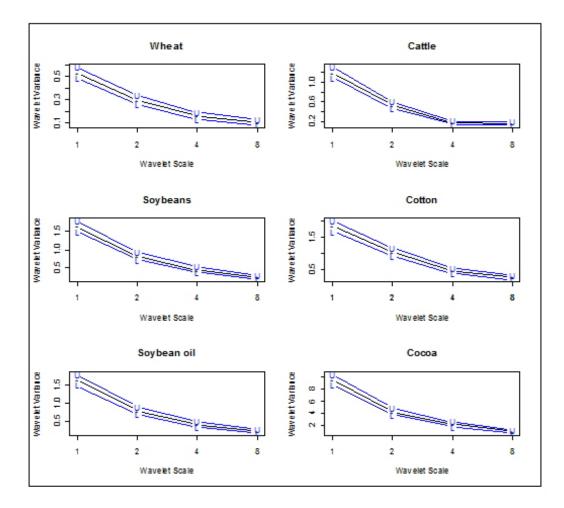


Figure 6. Estimated wavelet variance for Wheat, Cattle, Soybeans, Cotton, Soybean oil and Cocoa returns series.

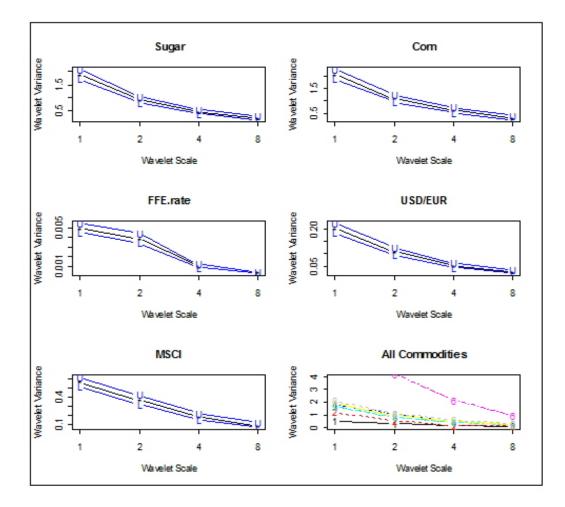


Figure 7. Estimated wavelet variance for Sugar, Corn, FFE.rate, USD/EUR, MSCI and all commodities returns series.

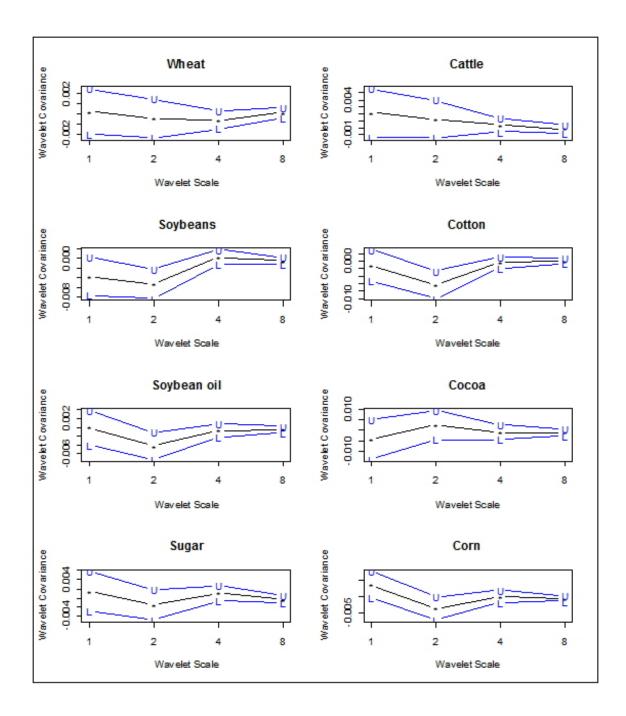


Figure 8. Estimated wavelet correlation between commodities returns and FFE rate

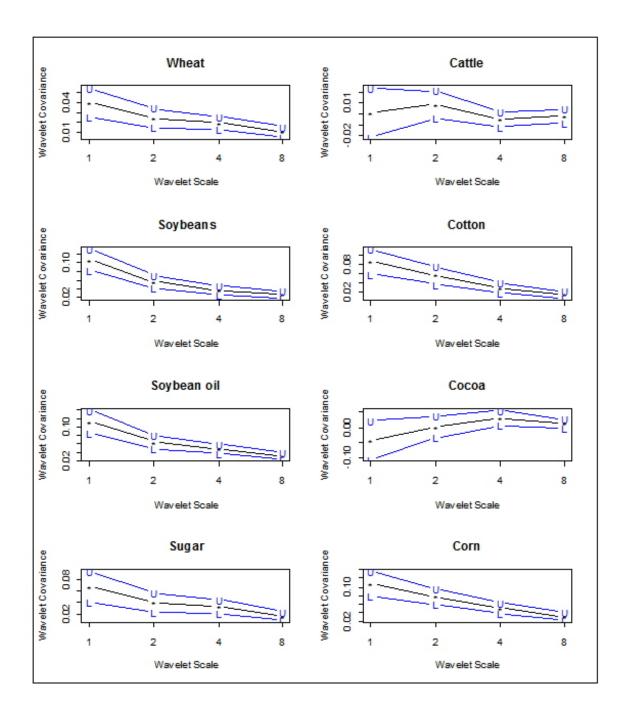


Figure 9. Estimated wavelet correlation between commodities returns and USD/EUR exchange rate

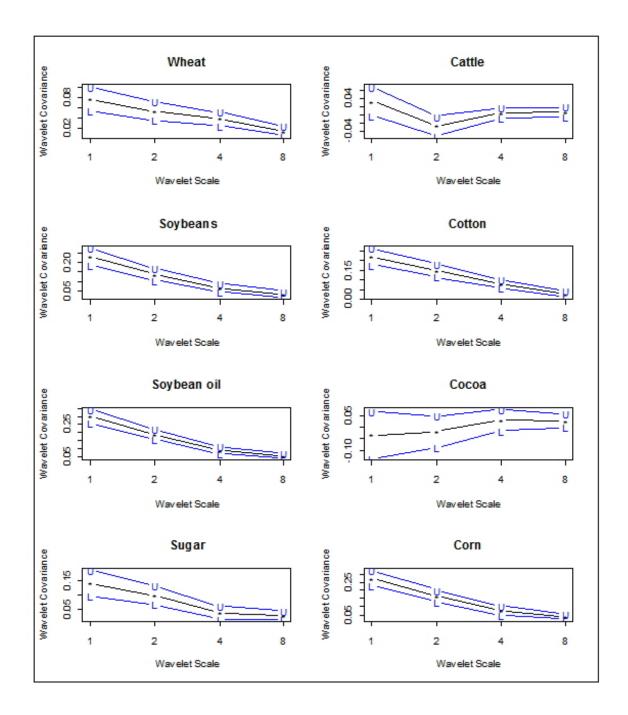


Figure 10. Estimated wavelet correlation between commodities returns and MSCI world market index